

# MIRRORS

## Specular and Matte Surfaces

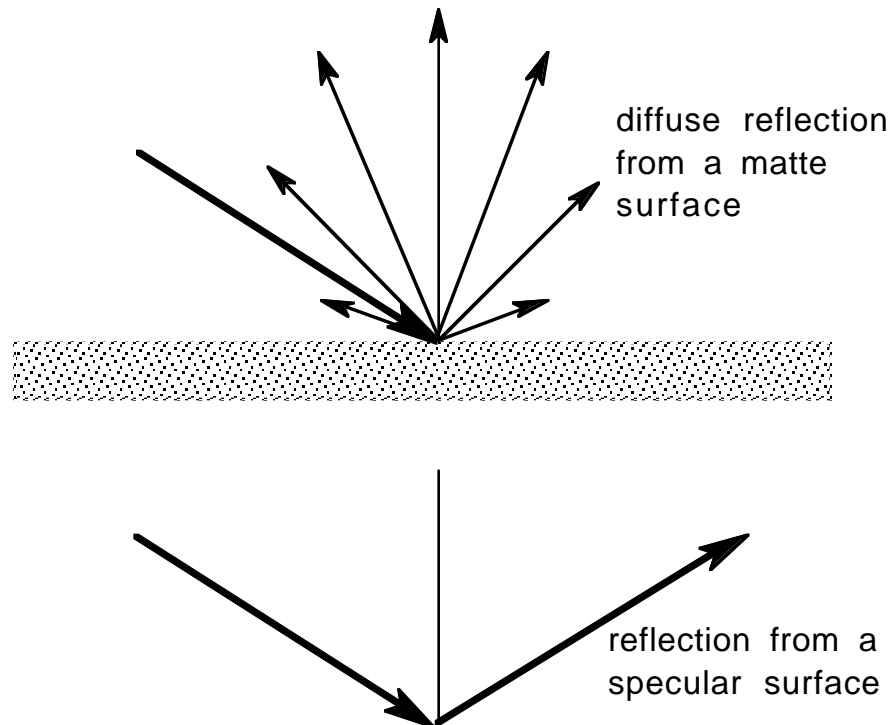
There are two kinds of surfaces which reflect incident light, specular surfaces and matte surfaces:



A matte surface is a rough surface which scatters an incident pencil of light in all directions. Examples would be a movie screen, a piece of paper or cloth, skin, in fact most objects in the real world.

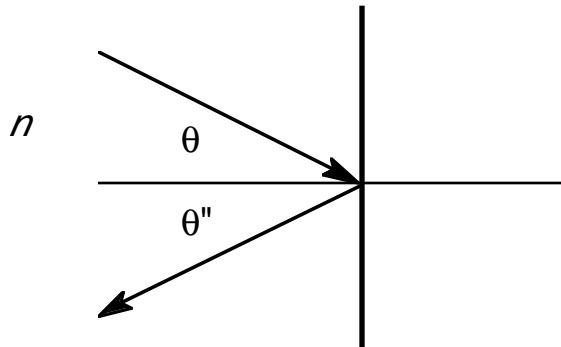


A specular surface is a smooth, shiny surface which reflects an incident pencil of light at an angle of reflection equal to the angle of incidence. Examples would be a mirror, a lens surface, chrome automobile trim.



## Image Formation by Mirrors

The law of reflection is simple, just the angle of incidence equals the angle of reflection as shown in the diagram below.



Recalling our sign convention, we should really write the law of reflection as  $\theta = -\theta''$ . Taking the sine of both sides of this equation,  $\sin\theta = \sin(-\theta'') = -\sin\theta''$ . Multiplying both sides by index  $n$ ,

$$n\sin\theta = -n\sin\theta''.$$

Compare this with Snell's law and we see that image space has an index  $n' = -n$ . In fact all our previously derived results for paraxial refracting surfaces can be applied again if we simply let  $n' = -n$  throughout. Specifically the fundamental paraxial equation becomes

$$L' = L + F, \\ h'/h = m = L/L'$$

where

$r = VC$	radius of mirror curvature
$l = VB$	object distance
$l' = V'B'$	image distance
$n$	index of object space
$-n$	index of image space
$L = n/l$	reduced vergence
$L' = -n/l'$	reduced vergence
$F = -2n/r$	surface power
$h = BQ$	object height
$h' = B'Q'$	image height
$m$	magnification

It's not necessary to know the index  $n$  unless dealing with combined

refracting and reflecting systems. Rewriting the paraxial equation with  $n$  factored out gives

$$1/f + 1/f' = 2/r.$$

Focal lengths are defined as before, so

$$F = -n/f = (-n)/f' = -2n/r,$$

so

$$f = f' = r/2.$$

**Example:** A concave mirror of radius of curvature 2m is used in a frame selection room. How much is the image of a patient one meter from the mirror magnified?

**Solution:** Here the object distance and incoming vergence is,  $L = -0.50\text{m}$ ,  $L = 1/(-0.50\text{m}) = -2.00\text{D}$ . The mirror power is  $F = -2/r = -2/(-2\text{m}) = +1.00\text{D}$ . The fundamental paraxial equation is  $L' = L + F = -2 + 1 = -1\text{D}$ , so  $f' = -1/L' = -1/(-1\text{D}) = +1.00\text{m}$ . Magnification is just  $m = L'/L = (-1)/(-2) = +0.50$ . So the image is erect and half the size of the object. It is also virtual and focused one meter behind the mirror.

## Geometric Ray Tracing for Mirrors

An infinite number of rays emerge from any luminous source. These rays travel in all directions. Because of the special properties of the focal points  $F$  and  $F'$  and the center of curvature,  $C$ , we may trace three of this infinitude of rays through the system. In fact any two rays would be enough to establish location and size of images. Use the following facts:



Rays travelling parallel to the axis before reflection are deviated so as to pass through (or have their extensions pass through) the secondary focal point  $F'$  after reflection.



Rays which intersect the primary focal point  $F$  (or have their extension intersect  $F$ ) before reflection travel parallel to the axis after reflection.

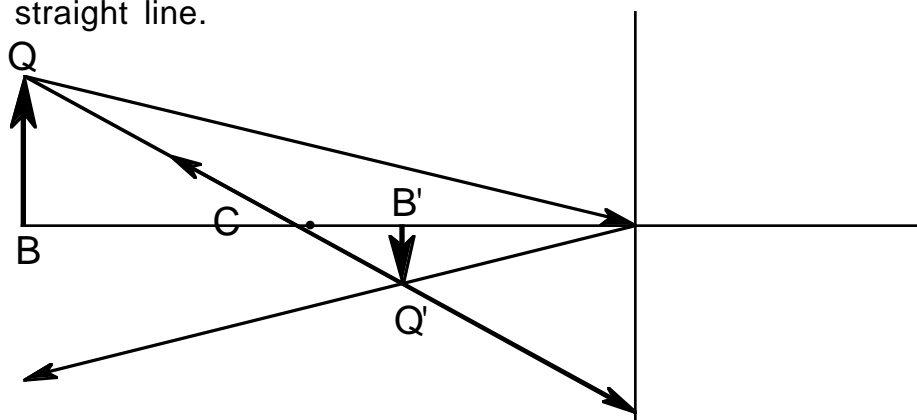


Rays directed toward (or from) the center of curvature of the reflecting surface strike the surface normally and so are reflected back along the same path by the reflecting surface.



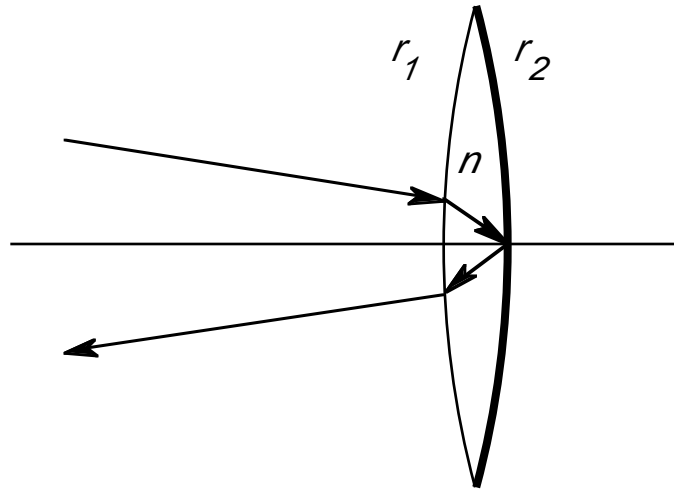
Rays directed toward (or from) the center of the reflecting surface are reflected at an angle equal to the angle of incidence.

The following diagram shows an example of using geometric ray tracing to construct the image of an object  $BQ$ . Note that the mirror is represented only by a straight line.



Reflecting and Refracting Systems

Lens mirror systems can be analyzed by the step-along method. One important system that's easy to understand is the lens with a silvered back surface in air.



As shown, rays are refracted at the front surface, bounce off the back surface, and then are refracted again at the front surface. The whole combination behaves like a mirror with power equal to that of the mirrored back surface plus twice the power of the refracting front surface. The power of the equivalent mirror,  $F_M$ , is then

$$F_M = 2F_1 - 2n/r_2 = 2(n-1)/r_1 - 2n/r_2.$$

The paraxial equation is the usual mirror equation

$$-1/l = 1/s + F_M.$$