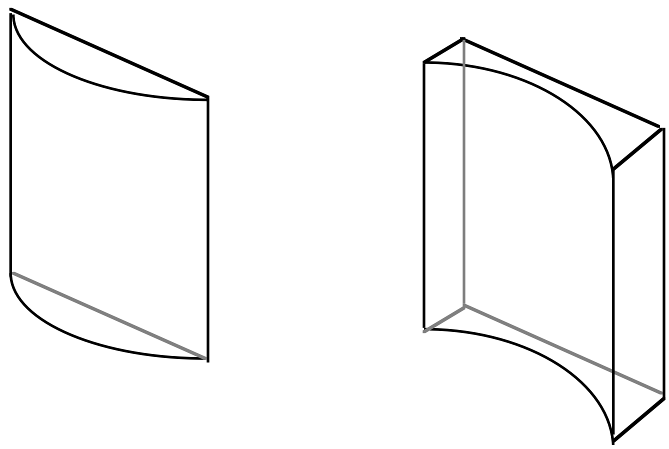
SPHEROCYLINDER LENSES

Cylinder Lenses

A cylinder lens is cut from the surface of a cylinder. Cylinder lenses can be concave or convex, as shown below.



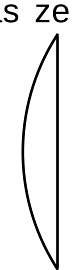
For the lenses shown, the horizontal meridian looks like a cross-section of a spherical lens and the vertical meridian looks like a plane wedge. The focusing of light in these meridians can be handled by the usual simple paraxial equations. To fully characterize a cylinder lens, not only the power of the cylinder F but also the angle of orientation of its axis α (90° in the figures above) must be specified.

Example: A convex crown glass hemi-cylinder of 5cm radius of curvature, negligible thickness, and vertical axis refracts light from a point 20cm away. What happens?

Solution: Find the power in each meridian of the cylindrical lens.

Horizontal meridian: The plane back surface has zero power so the power in the horizontal meridian is

$$F_H=(1.523-1)/0.05=+10.5D.$$



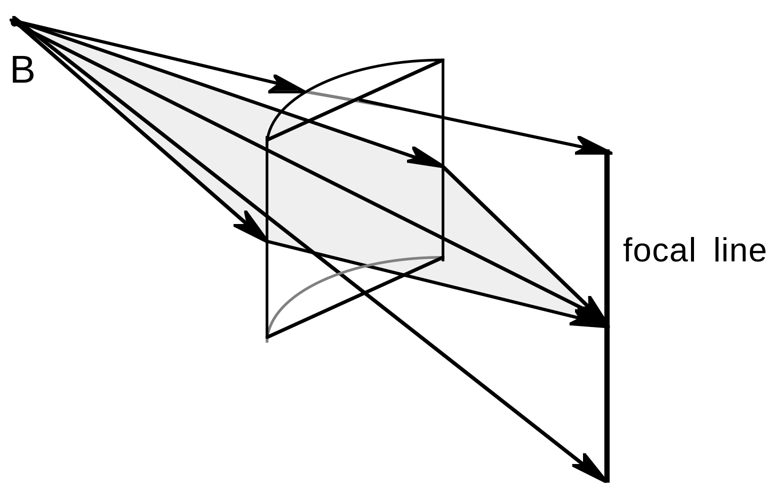
Vertical meridian: Both surfaces are plane so there is zero power in the vertical meridian.

The vergence of light reaching the lens is L=1/(-0.02)=-5.0D. So in the horizontal meridian the exiting vergence is

$$L'_H=F_H+L=+10.5-5.0=+5.5\text{D}$$
$$l'_H=1/L'_H=1/+5.5\text{D}=+18.3\text{cm}.$$

In the vertical meridian which has no power, l'_V=-20cm.

Seen in perspective, here is what would happen in the two meridians.

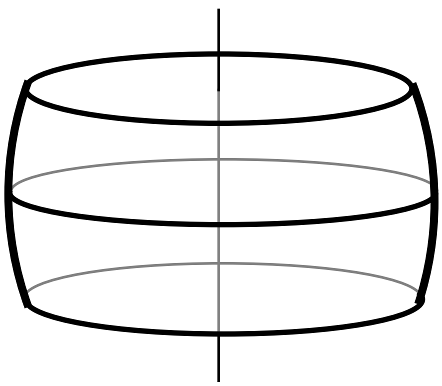


As can be seen, a <u>focal line</u> is formed 18cm behind the lens.

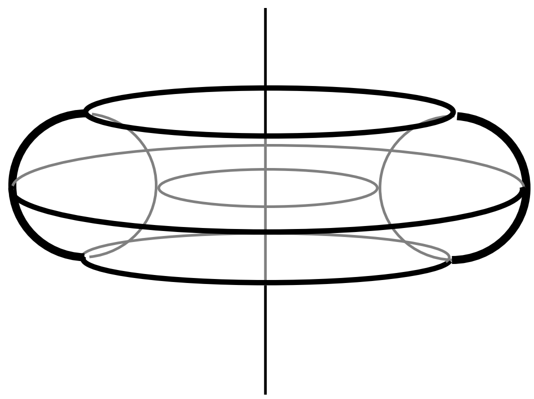
But what goes on in the oblique meridians? These oblique meridans are not spherical and have no real power. In a certain sense, however, they have a fictitious power $F_\theta=F\sin^2\theta$ where F is the cylinder power and θ the angle between the meridian in question and the cylinder axis.

Spherocylindrical Surfaces

Spherical surfaces have the same curvature in all meridians. Cylindrical surfaces are flat in one meridian and have maximum curvature in another, perpendicular, meridian. But lots of surfaces are curved in <u>all</u> meridians with different curvatures in all meridians, for example, donuts and watermelons. The simplest such surfaces are <u>toric surfaces</u> and toric surfaces are what's used in ophthalmic lenses. Astigmatism can be corrected with <u>cylinder</u> lenses. In practice it is corrected with lenses which have <u>toric</u> surfaces.



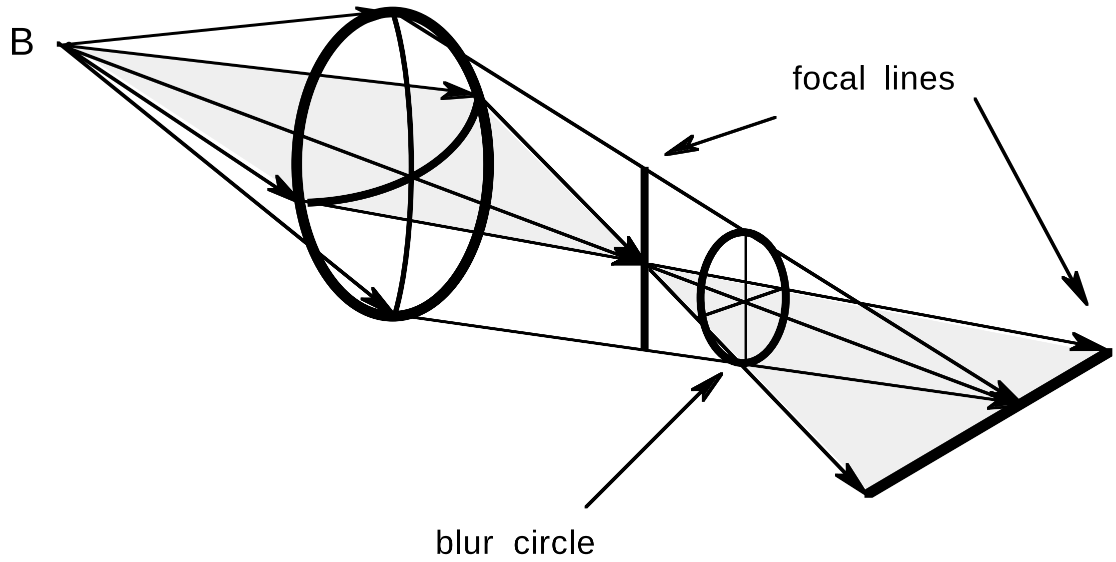
barrel form



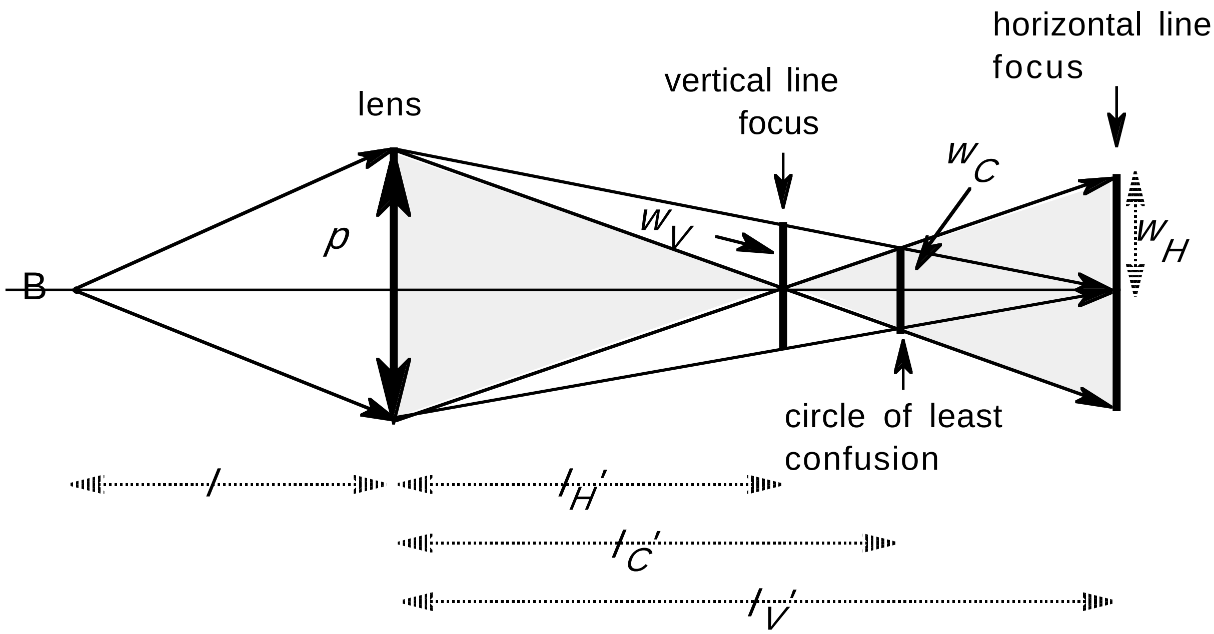
tire form

A toric surface is generated mathematically by rotating a circular arc around an axis that does **not** go through the center of the circle. (If we rotated it around the center of the circle we'd just get a sphere.) This generates surfaces in barrel form or tire form (donut form) as shown above.

A spherocylinder lens is usually made by grinding a spherical surface on one lens surface (the front surface for most spectacle lenses) and a toric surface for the other lens surface (the back surface for most spectacle lenses).



The diagram above shows a circular converging lens with a toric front surface bending light from a point source, B. As can be seen, the lens forms two perpendicular line foci with a blur circle in between. A receiving screen placed at any of these positions will show a line or a circle. At any other position, an ellipse will appear on the screen. Finding the positions of the line foci and circle of confusion is an interesting problem. It is solved by redrawing the horizontal and vertical line pencils in the plane of this page.



The positions of the two line foci can be calculated from the fundamental paraxial equation. By applying straightforward, but tedious, geometry to the figure above, it may be shown that the vergence of light forming the blur circle is the average of the vergences of light forming horizontal and vertical line foci, or in equation form,

$$1/l_C' = L_C' = (L_V' + L_H')/2 = (1/l_V' + 1/l_H')/2.$$

The linear interval between the two line foci, incidentally, is called the <u>interval of Sturm</u>. The half widths of the vertical and horizontal line foci, w_V and w_H, and the radius of the circle of least confusion, w_C, are

$$w_V = p(1 - L_V'/L_H'),$$
$$w_H = p(L_H'/L_V' - 1),$$
$$w_C = p[(L_H' - L_V')/(L_H' + L_V')].$$

Example: Find the positions of the line foci and blur circle and the interval of Sturm for a toric lens with +5.00D in the horizontal meridian and +8.00D in the vertical meridian when imaging a point object one meter in front of the lens.

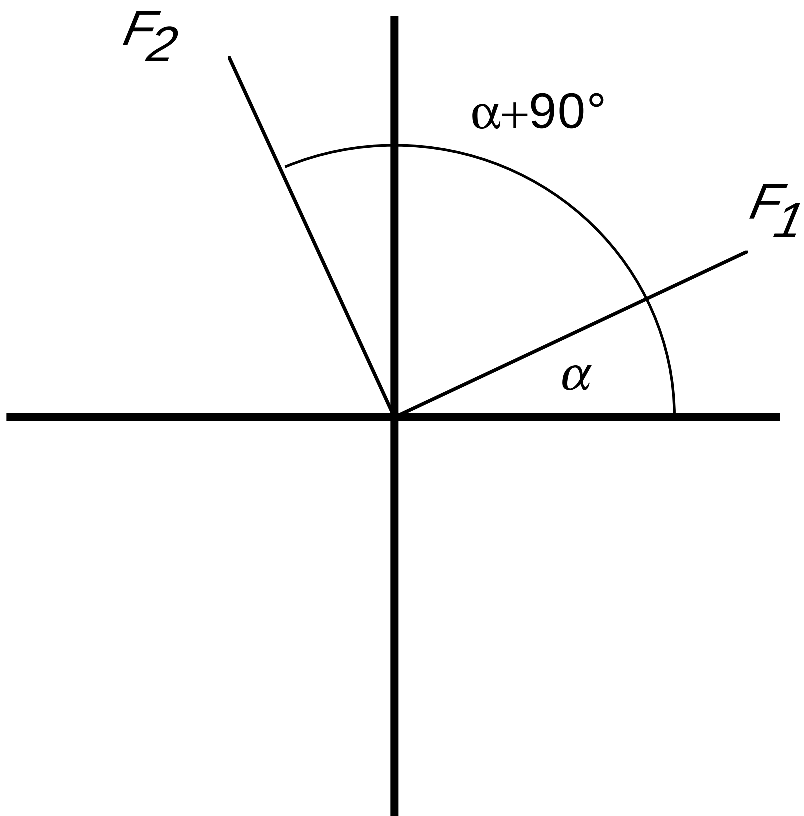
Solution: The incoming vergence is L=1/(-1m)=-1.00D. The outgoing vergence in the horizontal meridian is +5.00D-1.00D=+4.00D. This corresponds to a distance of 1/(+4)=+25cm, where the vertical line focus is formed. In the vertical meridian, outgoing vergence is +8.00D-1.00D=+7.00D. This corresponds to a distance of 1/(+7.00)=+14.3cm, where the horizontal line focus is formed. The interval of Sturm is just 25cm-14.3cm=10.7cm. The vergence going to the blur circle is (+4+7)/2=+5.5D so the blur circle is 1/(+5.5D)=+18.2cm behind the lens.

Astigmatic Imagery

An astigmatic system produces "smeared images", the amount and direction of the smear depending on the position of the receiving plane. At the horizontal line focus, for example, images will be smeared left to right while at the vertical line focus they will be smeared up and down. The "best" focus is usually taken to be in the plane of the circle of least confusion in which there is at least equal blurring in all directions. The "best" focus, however, might depend on the task and target. It is likely that patients with astigmatic eyes are constantly adjusting their focus along the interval of Sturm. That may account for the eyestrain of astigmats.

Ophthalmic Lenses

Spectacle lenses consist of one spherical surface and one toric surface. Prescriptions for spectacle lenses are, however, written as a superposition of spherical and cylindrical lenses. There are three ways to write a spectacle prescription; plus cylinder form (favored by M.D.'s), minus cylinder form (favored by optometrists), and cross cylinder form (used, sometimes, in fabrication labs). It's easiest to understand these forms and their relationship by reference to a power cross like the one below.



This cross represents a lens surface as viewed from the doctor's point of view, i.e. looking straight at the patient. The lens has two principal meridians (meridians of circular cross-section), with powers F_1 and F_2. Their orientations are shown on the diagram.

We could specify the prescription for the lens in the diagram by writing

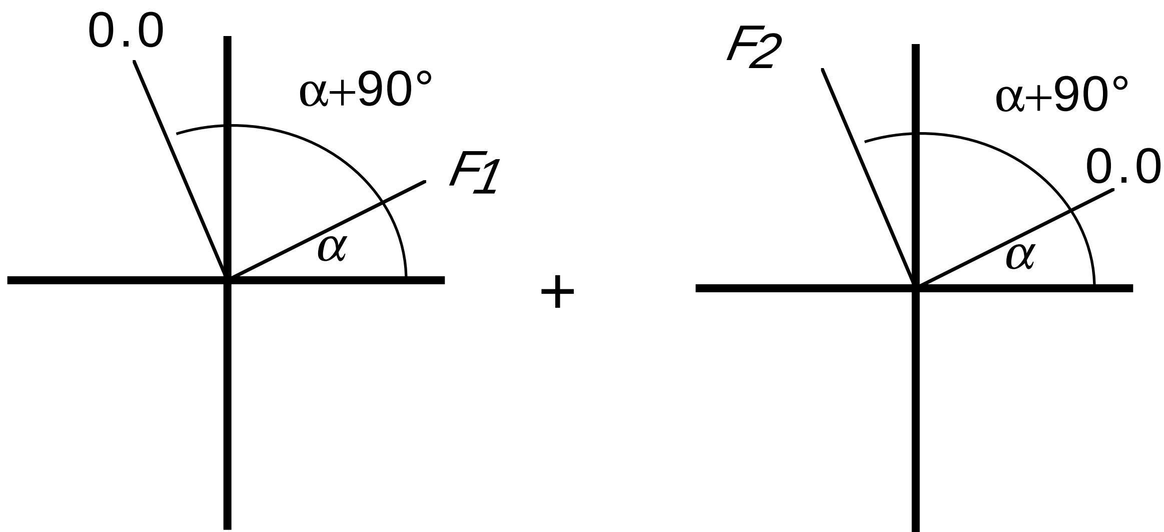
$$F_1@\alpha/F_2@(\alpha \pm 90°). \tag{1}$$

That is how keratometric findings (measurements of corneal curvature) are typically written, but lens powers are never written in this form. Recall that for a pure cylinder lens of power F_C and axis α the prescription can be written as $F_C \mathrm{x} \alpha$ or $F_C@(\alpha \pm 90°)$.

We can think of the lens in the diagram as a superposition of two cylinder lenses and write (1) as

$$F_S \mathrm{x} \alpha \,/ (F_S + F_C) \mathrm{x} (\alpha \pm 90°). \tag{2}$$

Equation (2) is the <u>cross cylinder form</u>. In diagram terms it sets up like below:



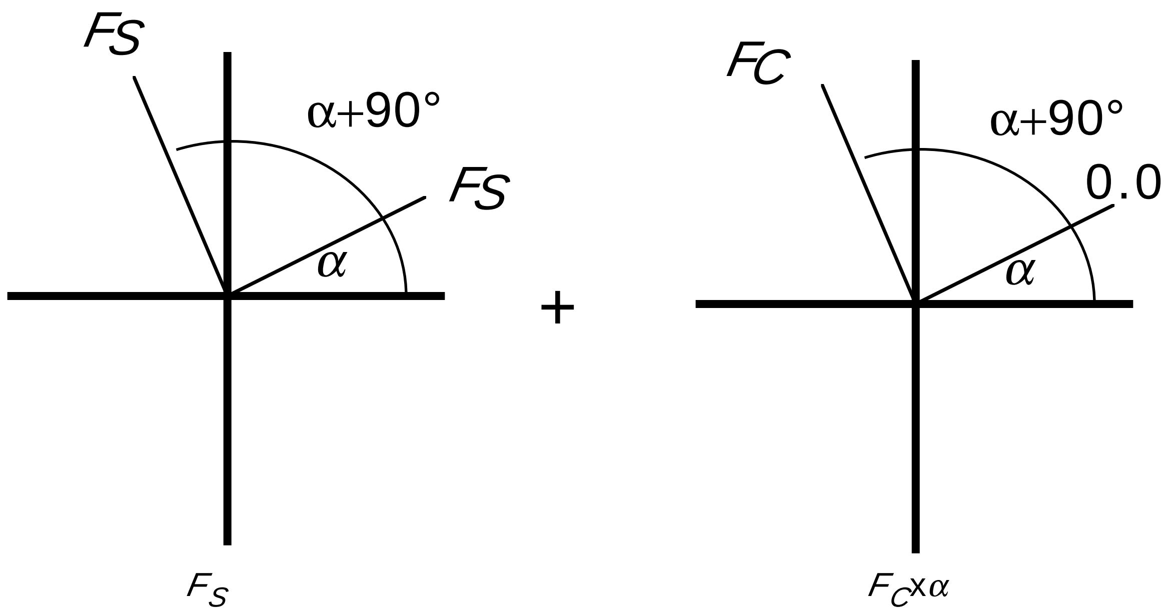
F_1 x(90°±α) $\qquad$ F_2 xα.

Since you can add sphero-cylinder lenses meridian by meridian, as long as the principal meridians are the same, you can see that this form gives the correct prescription.

Another, more common way of writing the prescription is in sphero-cylinder form in which the prescription is represented as a spherical lens of power F_S superimposed on a cylinder lens with power and axis $F_C x\alpha$. This is written in the form

$$F_S \,/\, F_C x\alpha. \tag{3}$$

In diagrams, equation (3) is represented as



The cylinder power can be positive or negative. The sign of the cylinder determines whether the prescription is in plus or minus cylinder form.

Now the problem is how to relate the sphero-cylinder form (3) to the crossed cylinder form (2). The key to doing this is in the diagrams. Clearly, by equating powers in corresponding meridians, $F_1 = F_S + F_C$ and $F_2 = F_S$. The cross cylinder form corresponding to the sphero-cylinder form is thus,

$$(F_S + F_C) x\alpha / F_S x(\alpha \pm 90°). \tag{4}$$

A given spectacle prescription can be written in any of these forms. The problem is converting among them. Here are three equivalent forms of a prescription.

$$F_S/F_C\text{x}\alpha$$
$$(F_S+F_C)/(-F_C)\text{x}(\alpha\pm 90°)$$
$$(F_S+F_C)\text{x}\alpha/F_S\text{x}(\alpha\pm 90°). \tag{5}$$

If $F_C>0$, these are, respectively, the plus cylinder, minus cylinder, and cross cylinder forms. Note that $\alpha\pm 90°$ is taken such that
$180°\geq\alpha\pm 90°>0$. In cross cylinder form it is customary to write the meridian for the smaller angle first.

Let's try it with some numerical examples. In each case there are three equivalent forms of the same prescription:

+3.00-1.00x030
+2.00+1.00x120
+2.00x030/+3.00x120

-1.50-2.50x100
-4.00+2.50x010
-1.50x010/-2.50x100

+1.00-4.25x025
-3.25+4.25x115
-3.25x025/+1.00x115

plano/+0.50x180
+0.50-0.50x090
0.00x090/+0.50x180

