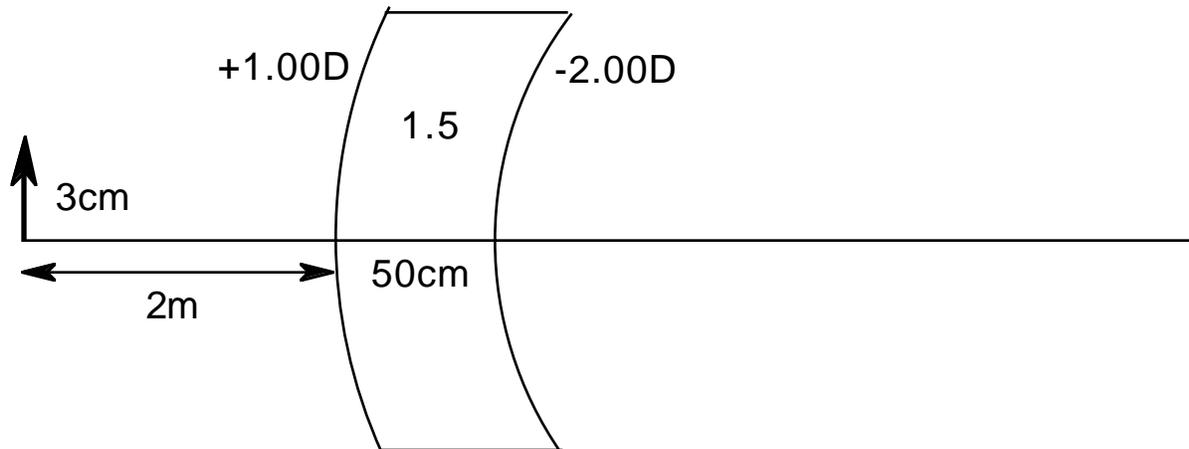


# STEP-ALONG METHOD

## Systems of Surfaces

Rays may be traced through a system by remembering the following credo: the image formed by one surface acts as the object for the next.

Example: Two surfaces of power  $F_1=+1.00D$  and  $F_2=-2.00D$  are separated 50cm by a medium of index 1.5. An object 3cm tall is placed before the first surface. Where is the image and what is its height?



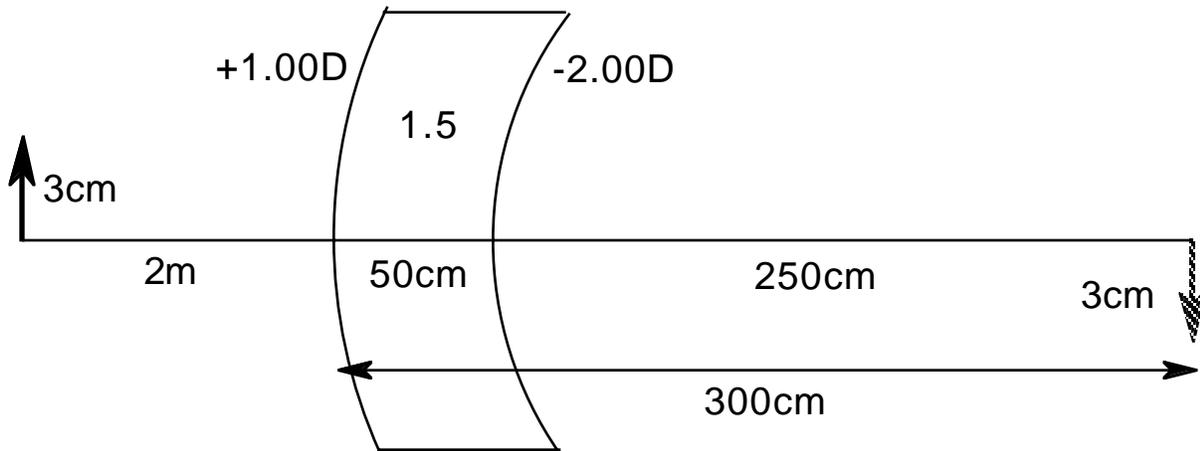
Solution: The vergence at the first surface is

$$L_1 = n_1/l_1 = 1.00/(-2.00\text{m}) = -0.50D.$$

From the fundamental paraxial equation,

$$\begin{aligned} L_1' &= L_1 + F_1 = -0.50 + 1.00 = +0.50D, \\ l_1' &= n_1'/L_1' = 1.5/0.5 = +3.00\text{m} = +300\text{cm} \\ h_1' &= m_1 h_1 = (L_1/L_1') h_1 = [(-0.5)/(0.5)](3\text{cm}) = -3\text{cm} \end{aligned}$$

So the image formed by the first surface is inverted, 3cm tall, and 300cm to the right of the first surface.



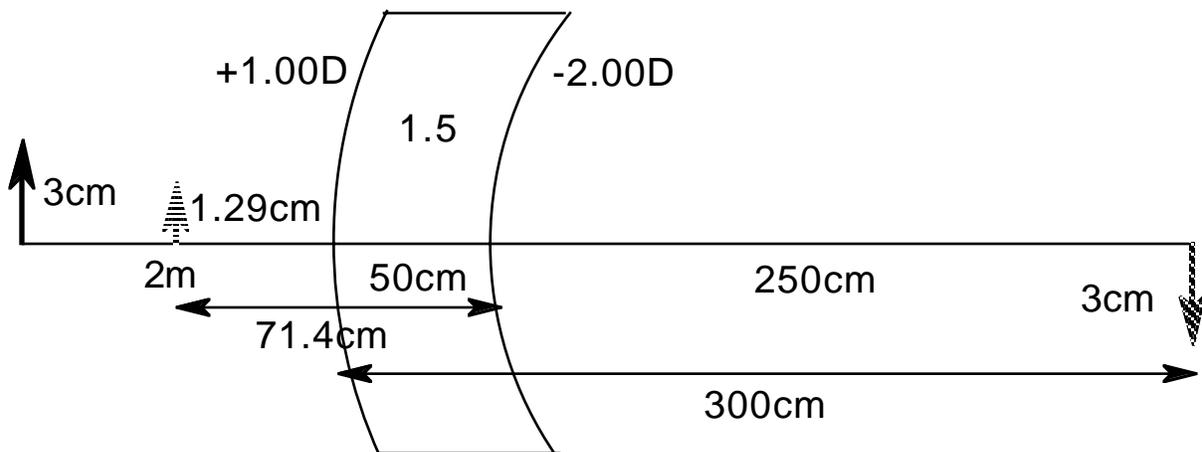
That image is shown in the diagram above. It becomes the object for the second surface so that  $l_2 = +250\text{cm}$ ,  $h_2 = -3\text{cm}$ . The incoming vergence is  $L_2 = n_2/l_2 = 1.50/(+2.50\text{m}) = +0.60\text{D}$ .

From the fundamental paraxial equation,

$$L_2' = L_2 + F_2 = +0.60 - 2.00 = -1.40\text{D},$$

$$l_2' = n_2'/L_2' = 1.00/(-1.40) = -0.714\text{m} = -71.4\text{cm}$$

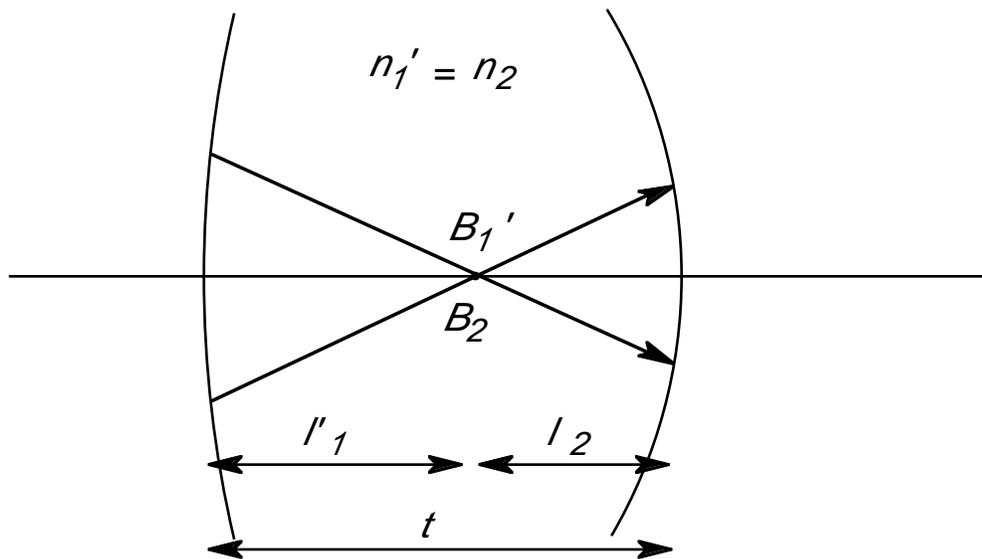
$$h_2' = m_2 h_2 = (L_2/L_2') h_2 = [(0.60)/(-1.40)](-3\text{cm}) = +1.29\text{cm}$$



That final image is virtual, erect, 1.29cm tall, and 71.4cm to the left of the second surface. It is shown in the diagram above.

The procedure above is called the step along method. It can be applied to any centered optical system, but it's quite tedious.

We can save a little time in such calculations by not converting vergences to object distances.



From geometry and the definition of vergence,

$$l_1' = n_1' / L_1'$$

$$l_2 = l_1' - t$$

$$L_2 = n_2 / l_2 = n_2 / (l_1' - t) = n_2 / (n_1' / L_1' - t)$$

or finally

$$L_2 = L_1' [1 - (t/n_1')] L_1$$

Note also that the total magnification after refraction by a series of surfaces is just the product of the magnification at each surface, i.e.

$$m = m_1 m_2 m_3 \dots = (L_1 / L_1') (L_2 / L_2') (L_3 / L_3') \dots$$

Let's rework the previous problem using these last two equations.

Example Redone:

At the first surface,

$$L_1' = L_1 + F_1 = -0.50 + 1.00 = +0.50\text{D},$$

$$m_1 = (L_1/L_1') = (-0.5)/(0.5) = -1.$$

The vergence of light reaching the second surface is

$$L_2 = L_1' [1 - (t/n_1')] = +0.5 [1 - (0.5/1.5)(+0.5)] = +0.6.$$

Using the fundamental paraxial equation,

$$L_2 = L_2 + F_2 = +0.60 - 2.00 = -1.40\text{D},$$

$$l_2' = n_2'/L_2' = 1.00/(-1.40) = -0.714\text{m} = -71.4\text{cm}$$

$$m_2 = (L_2/L_2') = (0.60)/(-1.40) = -0.429.$$

The total magnification is  $m = m_1 m_2 = (-1.00)(-0.429) = +0.429$   
 so the image height is  $mh_1 = (+0.429)(3\text{cm}) = +1.29\text{cm}$ .

Thus the answers are the same as before, but the calculation is a bit simpler.

