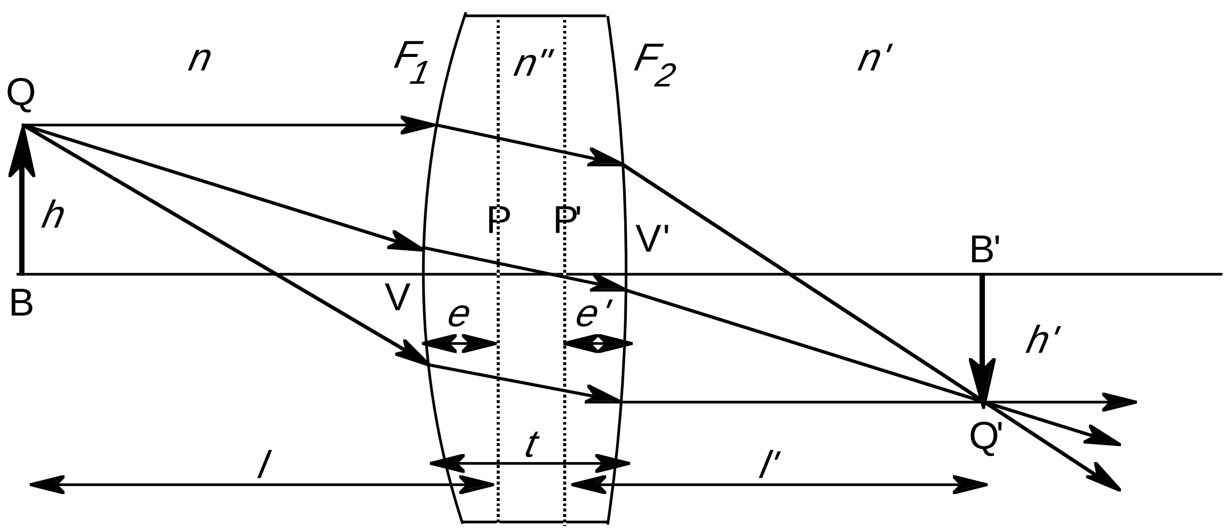
THICK LENSES and LENS SYSTEMS

Principal Points of a Thick Lens

Any optical system may be analysed with the step-along system, but it's really tedious and time consuming. When we worked with a thin lens we found that we could use the same fundamental paraxial equation we used for a thin lens if we treated the lens position as the reverence point V instead of the surface vertex and let the thin lens power be the algebraic sum of the power of the two lens surfaces. Wouldn't it be wonderful if we could do the same thing with a thick lens?

Well, we can't. Not quite. But we can come close. Consider the thick lens shown below.It has thickness t, index n'', and surface powers F_1 and F_2.



Object and image space have indices n and n', respectively. If we want to find the image of object BQ we *could* use the step along method, but alternatively we can write the usual fundamental paraxial equations,

$$L'=L+F, \text{ where } L=n/l, \text{ and } L'=n'/l',$$
$$m=h'/h=L/L'.$$

if we just reinterpret the quantities a bit. It turns out that the optical power of the thick lens, called its <u>equivalent power</u> is given by

$$F=F_1+F_2-(t/n'')F_1F_2.$$

The object and image distances are measured from a pair of reference points called the principal points which lie on the optic axis in the principal planes. The principal planes are labelled P and P' in the diagram above. The location of the principal planes of a thick lens may be shown to be

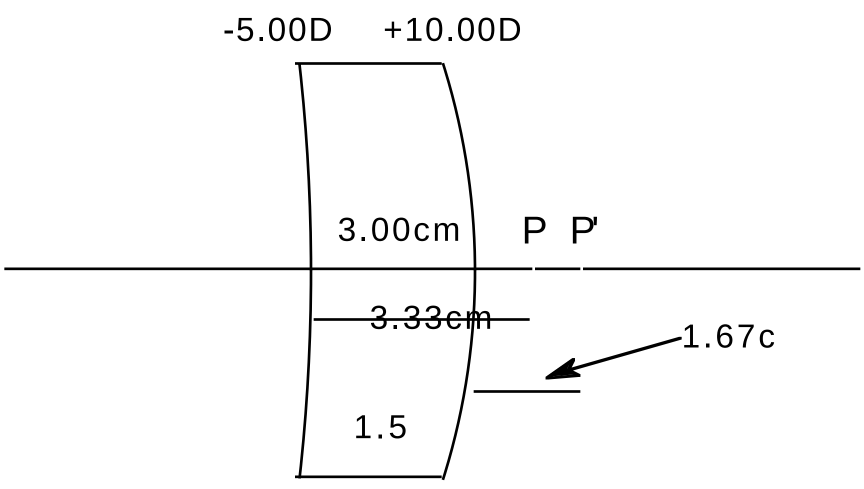
$$e=VP=(nt/n'')(F_2/F),$$
$$e'=V'P'=-(n't/n'')(F_1/F).$$

So where did these equations come from? Actually there are various ways, my favorite being a matrix optics formalism. But it doesn't really matter right now.

Example: The front surface of a thick lens in air has -5.00D power and the back surface has +10.00D power. The lens is 3cm thick and has index 1.5. Find the equivalent power of the lens and locate it principal planes.

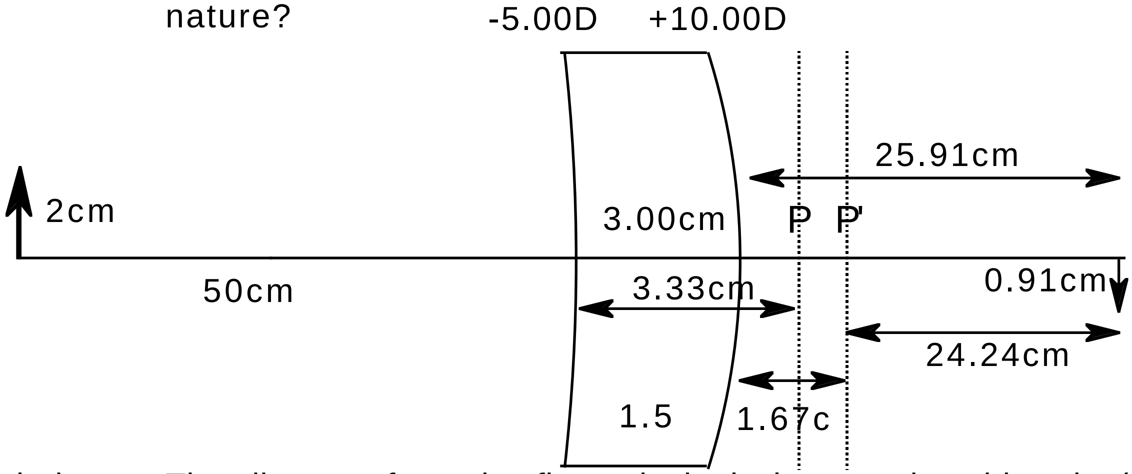
Solution: From the equations,

$$F=-5+10-(0.03)(-5)(10)/1.5=+6.00D,$$
$$e=(0.03)(10)/[(1.5)(6)]=3.33cm,$$
$$e'=-(0.03)(-5)/[(1.5)(6)]=1.67cm.$$



The positions of the principal planes is shown in the diagram.

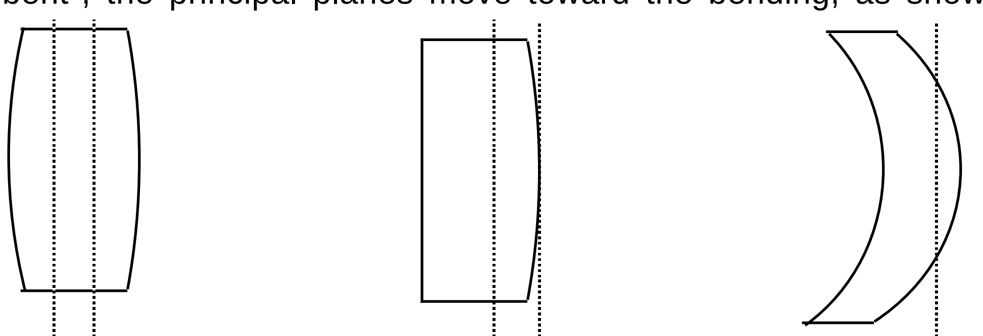
Example: An object 2cm tall is 50cm in front of the lens in the previous example. Where is its image, how tall is it, and what is its nature?



Solution: The distance from the first principal plane to the object is l=-50cm-3.33cm=-53.33cm so L=1/l=1/(-.5333)=-1.875D. Using the fundamental paraxial equation, the vergence of the image with respect to the second principal plane is L'=-1.875D+6.00D=+4.125D so that the image distance is l'=1/(+4.125)=+24.24cm measured from the second principal plane. The distance of the image from the back surface of the lens is 24.24cm+1.67cm=+25.9cm.

The magnification of the image is m=L/L'=-1.875/4.125=-0.455 and the image height is h'=mh=(=0.455)(2cm)=-0.91cm. The image is real and inverted.

By playing with the equations for e and e' we can get a few general ideas about principal plane position. For example, for an equiconvex or equiconcave lens, the principal planes are located symmetrically at about 1/3 and 2/3 the lens thickness. For a plano-convex or plano concave lens, the one principal plane is tangent to the surface with power. As a lens form is "bent", the principal planes move toward the bending, as shown below.



Focal Lengths of a Thick Lens

Analogous to the focal lengths of other optical systems, the focal lengths of a thick lens, f and f', are defined

$$F=-n/f=n'/f'.$$

The primary and secondary focal points may be located from f=PF and f'=P'F'.

We also define primary and secondary back vertex focal lengths f_V =VF and f_V' =V'F'. We can define corresponding front and back vertex powers from

$$\text{front vertex power}=F_V=-n/f_V,$$
$$\text{back vertex power}=F_V'=n'/f_V'.$$

The back vertex power is of considerable importance in ophthalmic optics. By using the step along system we can write these vertex powers in terms of the surface powers, lens thickness t, and lens index n'' as

$$\text{front vertex power}=F_V=F_1+F_2/[1-(t/n'')F_2]=F/[1-(t/n'')F_2],$$
$$\text{back vertex power}=F_V=F_2+F_1/[1-(t/n'')F_1]=F/[1-(t/n'')F_1].$$

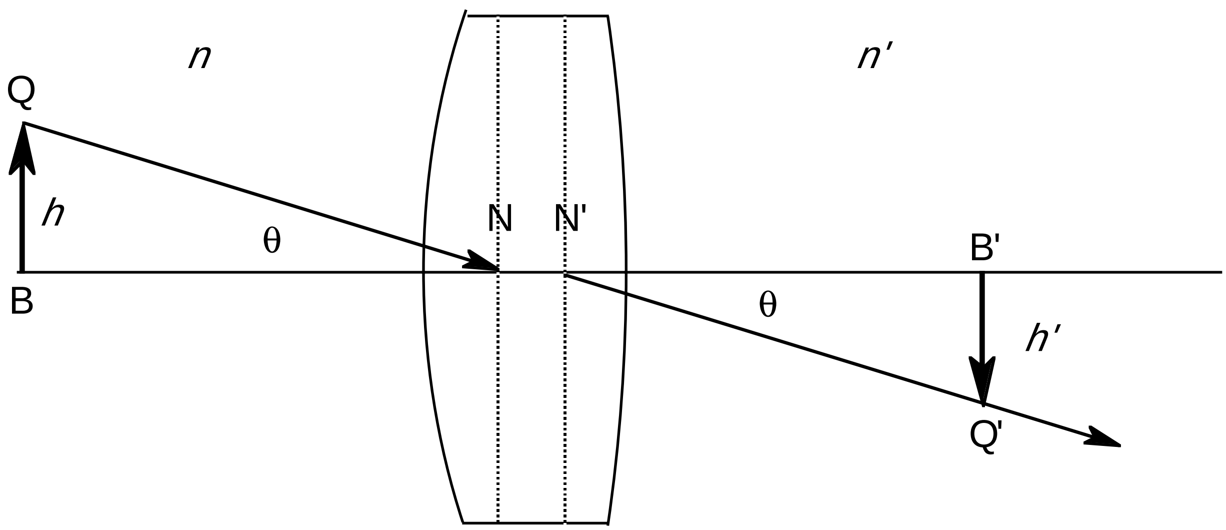
Example: Find the front and back vertex powers of the lens in the previous two examples.

Solution: Recalling that F=+6.00D, F_1=-5.00D, F_2=+10.00, n=1.5, and t=0.03m, plugging in gives

$$F_V=+6/[1-(0.03/1.5)(+10)=+7.5\text{D},$$
$$F_V=+6/[1-(0.03/1.5)(-5)]=+5.45\text{D}.$$

Nodal Points of a Thick Lens

It can be shown that there is a point in any optical system such that a ray aimed toward that point emerges translated but not deviated. This is shown in the following diagram.



As shown, the ray aimed at point N emerges from the optical system *as if* it came from point N'. The points N and N' are called the primary and secondary nodal points of the lens. If $n=n'$ as it would for a lens in air, the nodal points would coincide with the corresponding principal points. Otherwise they can be found by noting that FN=f' and N'F'=-f. The separation of the nodal points is always the same as the separation of the principal point, NN'=PP'. They behave similarly to the center of curvature of a single refracting surface or the optical center of a thin lens.

The two focal points, the two principal points, and the two nodal points are called the <u>cardinal points</u> of the thick lens.

Geometric Ray Tracing

If four of the cardinal points of a system are known it is possible to do geometric ray tracing. The following rules show how to construct the rays. Note that these are construction rays, <u>not</u> actual rays.

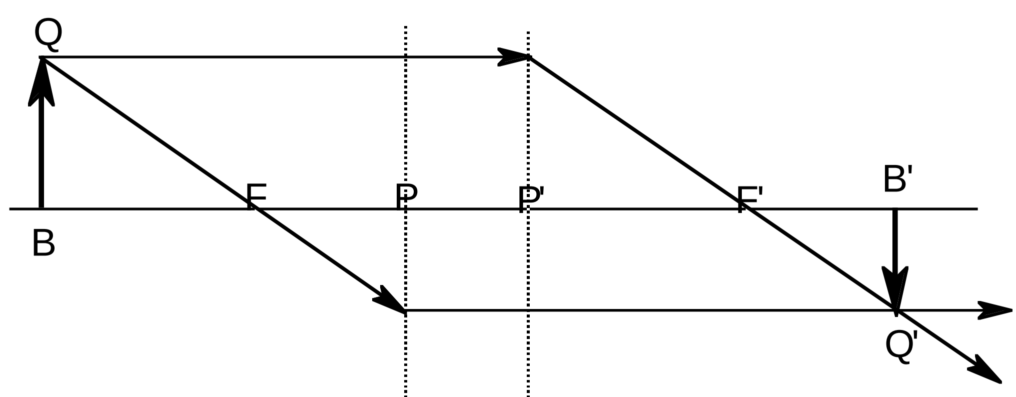
☞ Rays travel parallel to the axis between the primary and seconary principal point.

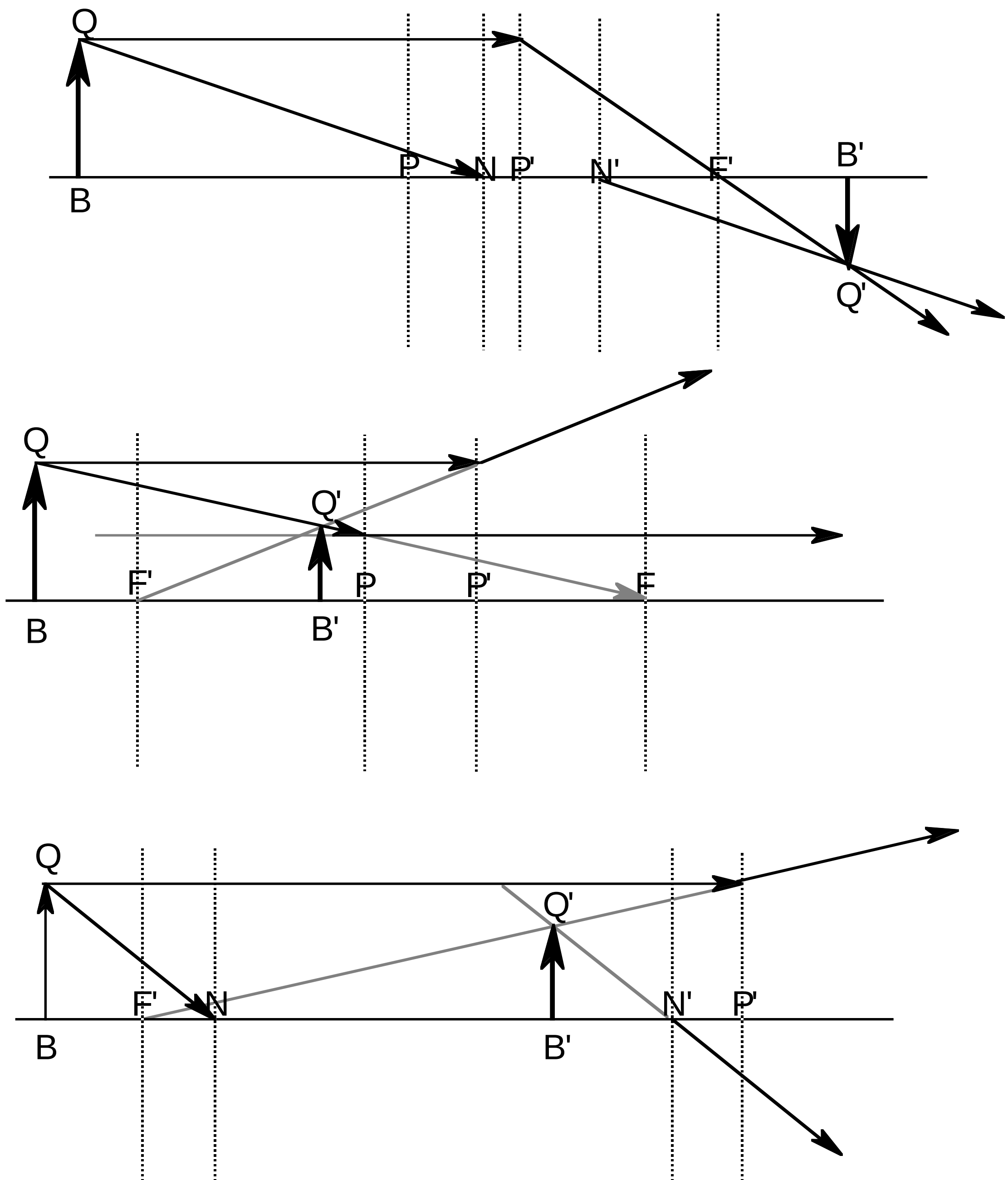
☞ Rays travelling parallel to the axis before refraction are deviated so as to pass through (or have their extensions pass through) the secondary focal point F' after refraction.

☞ Rays which intersect the primary focal point F (or have their extension intersect F) before refraction travel parallel to the axis after refraction.

☞ Rays directed toward (or from) the primary nodal point are emerge undeviated from the secondary nodal point.

Some examples of geometric ray tracing through a thick lens are shown in the following diagrams.





An object at the first principal plane is conjugate to in image of identical size at the other principal plane. For this reason these are sometimes refered to as <u>planes of unit magnification</u>.

General Optical Systems

Cardinal points may be used to describe *any* optical system. Once the cardinal points are known, all the information necessary to find images is known. But calculating the positions of the cardinal points for any optical system with more surfaces than a thick lens is a complicated proposition. There are several algorithms. One excellent one involves matrix operators which make the problem easy to set up. Doing actual calculations, however, is still very time consuming.

Another way of finding the cardinal points of a general optical system is to combine the elements of the system two by two. As a simple case, consider a pair of thin lenses in air. If the lens powers are F_1 and F_2 and the separation of the lenses is t, the equivalent power of the system is

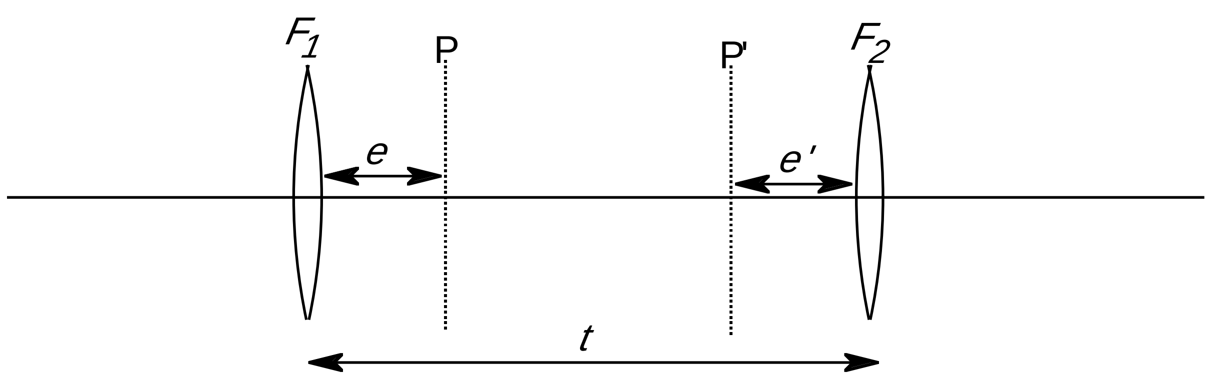
$$F=F_1+F_2-tF_1F_2.$$

The distance from the first lens to the primary principal point is

$$e=tF_2/F,$$

and the distance from the second lens to the secondary principal point is

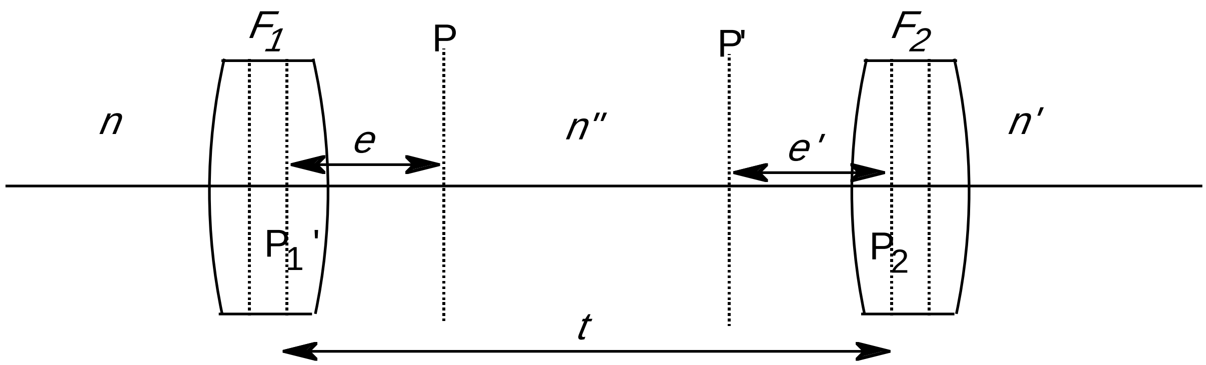
$$e'=-tF_1/F.$$



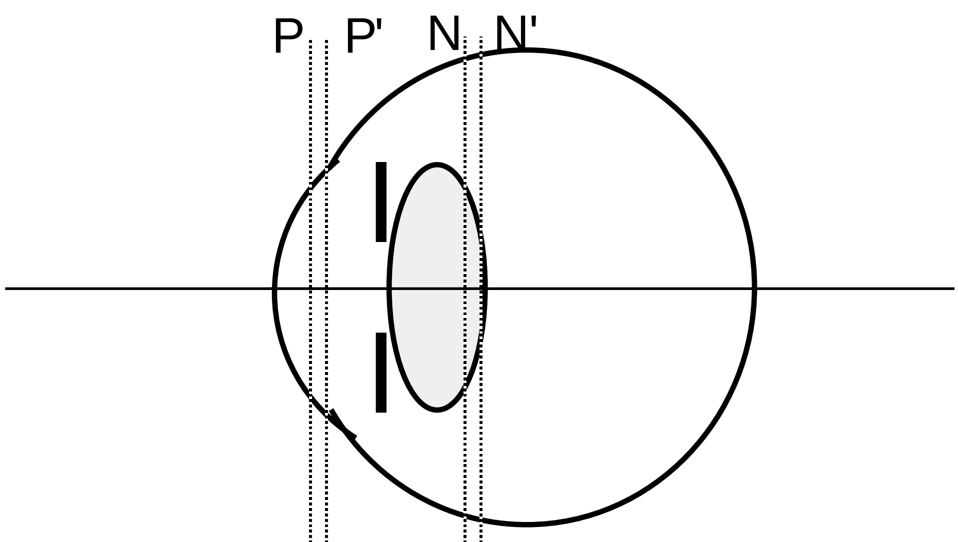
If we have two optical systems and we know their equivalent powers F_1 and F_2 and the positions of the principal points of both, as well as the indices in object space n, image space n', and the space between the systems n'', the equations above generalize to

$$F = F_1 + F_2 - (t/n'')F_1F_2,$$
$$e = n(t/n'')F_2/F,$$
$$e' = -n'(t/n'')F_1/F.$$

where t is now the distance between the secondary principal point of the first system and the primary principal point of the second system, and e and e' are measured with respect to those same principal points, as indicated in the diagram below.



Much of the time someone else does the calculations for us. Here, for example is the layout of the cardinal points in a schematic eye, a model of the human eye.



Note that the two principal points almost coincide, as do the two nodal points. Note also that the two nodal points are at the back of the crystalline lens. Some optometrists have used that to explain why posterior capsular cataracts are so devastating to vision "since all the rays go through the nodal points". Of course they don't, really. Real rays bounce from surface to surface, unaware of the existence of principal or nodal points. To see the distinction between real and construction rays, let's finish off with one more look at the thick lens we started with. The solid rays in the diagram are the actual rays traversing the system. The Construction rays are shown in gray.