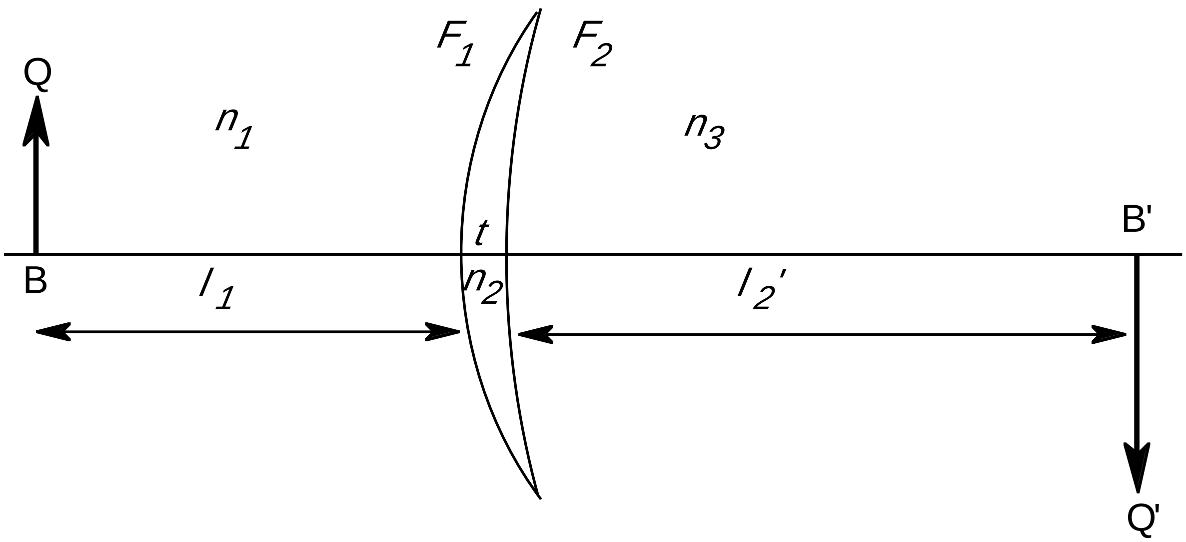
THIN LENSES

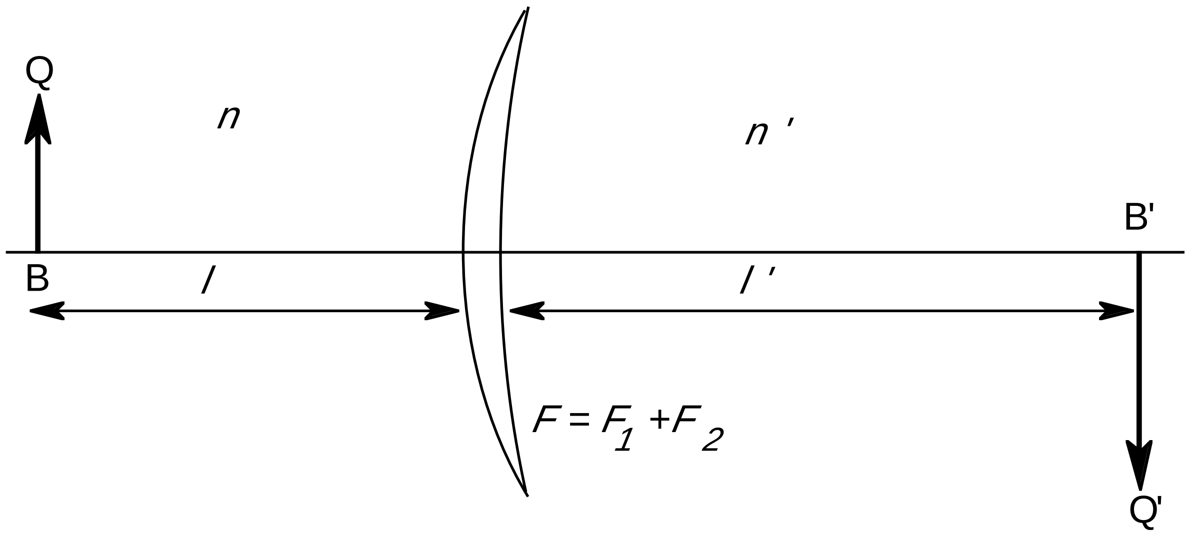
Fundamental Paraxial Equation for Thin Lenses

A thin lens is one for which thickness is "negligibly" small and may be ignored. Thin lenses are the most important optical entity in ophthalmic optics.

Let's use the step-along method to find the image of an object formed by a thin lens.



The vergence of light striking the first surface is $L_1 = n_1/l_1$. From the fundamental paraxial equation, $L_1' = L_1 + F_1$. The vergence of light reaching the second surface is $L_2 = L_1'/[1-(t/n_2)L_1'] \cong L_1'$. The last comes from the fact that t is so small. Applying the fundamental paraxial equation to the second surface, $L_2' = L_2 + F_2 = L_1 + F_1 + F_2$. Magnification is given by $m = m_1 m_2 = (L_1/L_1')(L_2/L_2')$ but since $L_2 \cong L_1'$ this becomes $m = L_1/L_2'$.



These last two equations tell us that the thin lens acts like a single optical entity with power $F=F_1+F_2$ which obeys the fundamental paraxial equation

$$L'=L+F$$
$$m=L/L'$$

where $L=n/l$ is the vergence of light reaching the first surface of the lens and $L'=n'/l'$ is the vergence of light leaving the second surface.

We can define primary and secondary focal lengths just as with a single refracting surface to get the equations

$$f'=n/F$$
$$f=-n'/F.$$

Note that in realistic problems, the lens is invariably in air so that $n=n'=1.00$. In that case or in any case where object and image space have the same indices, $f'=-f$.

Geometric Ray Tracing

As with a single refracting surface, geometric ray tracing is useful in visualizing the solution to optical problems involving thin lenses and inverifying, at least qualitatively, what's going on.

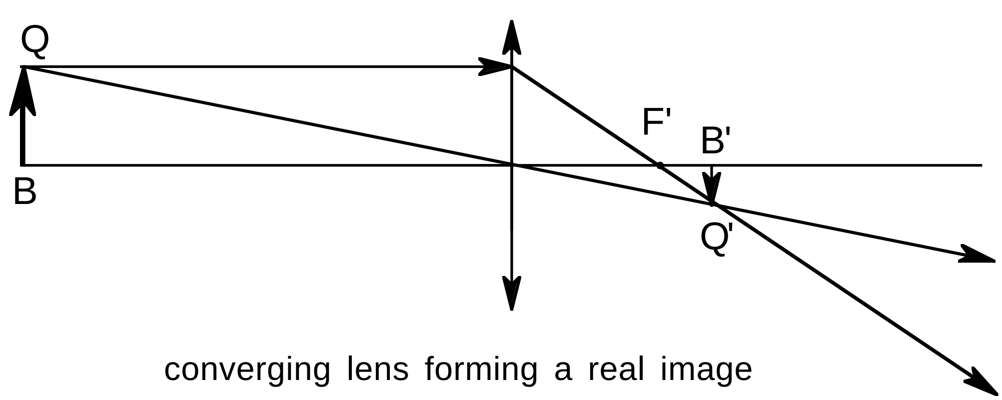
An infinite number of rays emerge from any luminous source. These rays travel in all directions. Because of the special properties of the focal points F and F' and the ray through the center of the lens, we may trace three of this infinitude of rays through the system. In fact any two rays would be enough to establish location and size of images. Use the following rules to trace rays through a thin lens in air:

☞ Rays travelling parallel to the axis before refraction are deviated so as to pass through (or have their extensions pass through) the secondary focal point F' after refraction.

☞ Rays which intersect the primary focal point F (or have their extension intersect F) before refraction travel parallel to the axis after refraction.

☞ Rays directed toward (or from) the center of the lens are undeviated by the refracting surface.

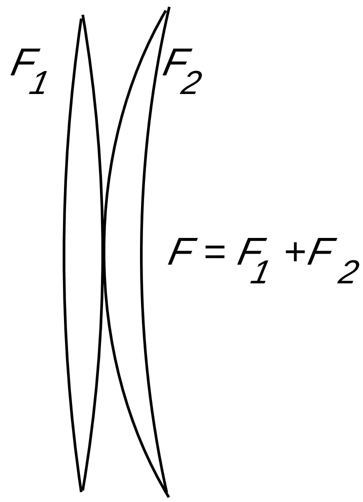
The following diagram shows an example of using geometric ray tracing to construct the image of an object BQ. Note that the refracting surface is represented only by a straight line.



converging lens forming a real image

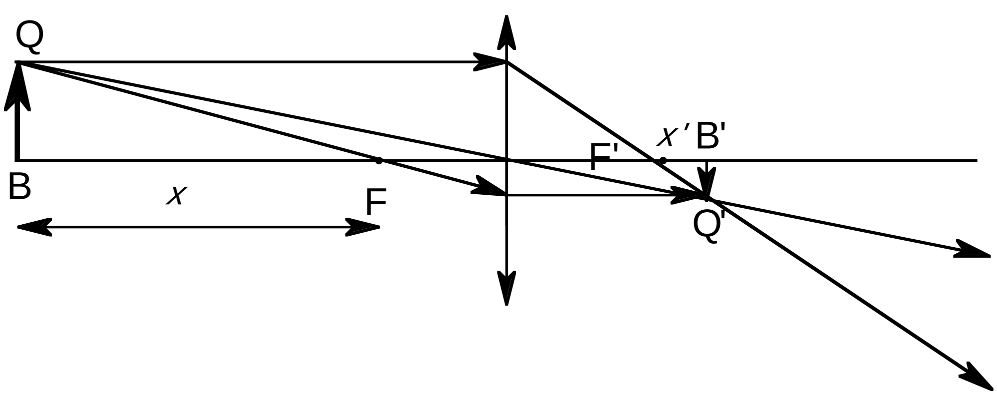
Superposition of Thin Lenses

Suppose two thin lenses of power F_1 and F_2 are placed in contact. The combination of lenses behaves optically like a single thin lens of power $F = F_1 + F_2$.



Newton's Relation

Another way of writing the fundamental paraxial equation is Newton's relation in which object and image distances are given in terms of the distances from the primary and secondary principle points, respectively.



The extra-focal distances x=FB and x' =F'B' are related by Newton's relation

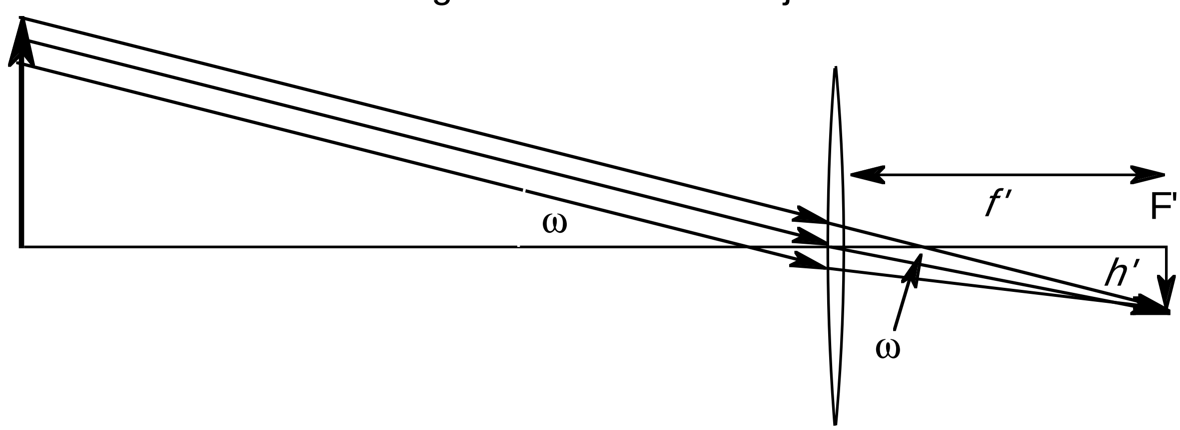
$$xx' = ff'.$$

In terms of the extra-focal distances, magnification is

$$m = -f/x = -x'/f'.$$

Newton's relation may be derived in straightforward, if tedious, fashion from the fundamental paraxial equation. It is useful in understanding the operation of the lensometer, but not much else.

Image of a Remote Object

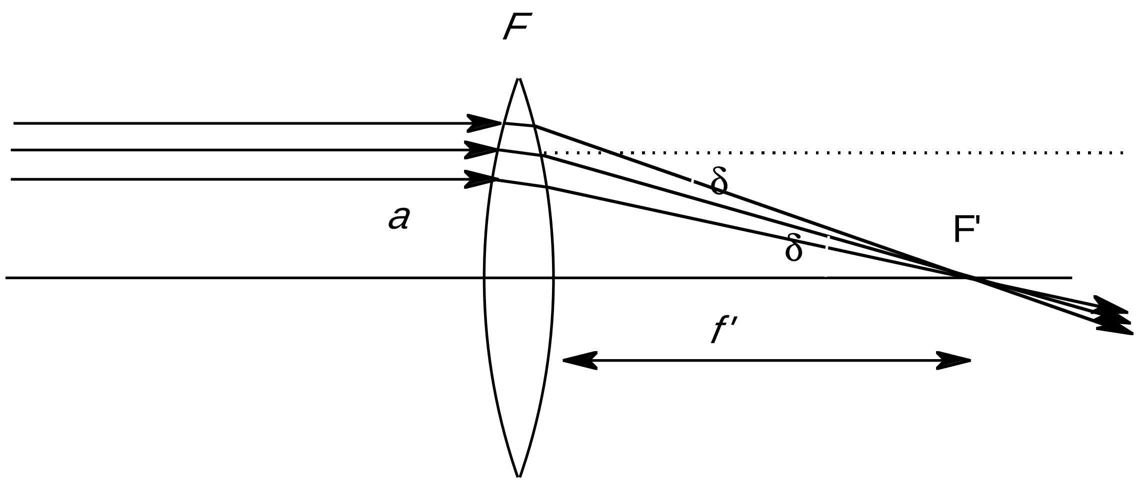


Suppose a remote object subtends angle ω in object space in front of a thin lens. It's image is formed at the secondary focal point a distance f' behind the lens. The height of the image may be determined from the similar triangle in image space to be

$$h' = -f'\omega.$$

Prismatic Effects in Lenses

We can think of a lens as a stack of prisms, each with slightly different power. Not surprisingly, then, extra-axial points behave as if they had a prism power as well as a lens power. We can calculate that prism power from the diagram below.



Consider a parallel bundle of rays striking a lens of power *F* a distance *a* from the optical axis. Rays converge to the secondary focal point. The ray bundle is rotated through an angle δ where $\delta = -a/f' = -aF$. To get the prismatic deviation, remember that prism power Δ is one hundred times the angle of deviation in radians so

$$\Delta = 100\delta = -100aF.$$

This is the famous Prentice's Rule. If we give *a* in centimeters we can drop the factor of 100 and write simply

$$\Delta = -a(\text{cm})F.$$

Example: What is the prism 2 centimeters above the optic axis for a +3.00D lens?

Solution: Δ=-(2cm)(+3.00D)=-6p.d., 6p.d. base down.

Thin Lens with an Aperture

Aperture effects are among the most vexed topics in all of geometric optics. Basically apertures can do four things:

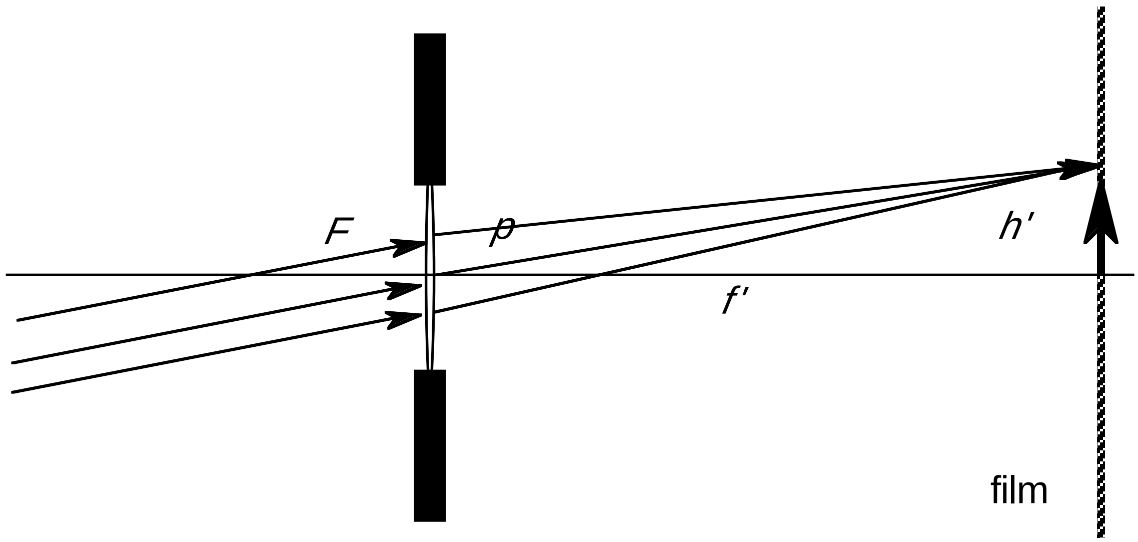
☞ Apertures control the amount of light that gets into an optical system and hence the intensity of the image formed by the system.

☞ Apertures determine the field of view of an optical system.

☞ Apertures determine the amount of blur of an out-of-focus image and hence the depth of field and focus of an optical system.

☞ Apertures determine the diffraction limit on resolution of the optical system.

The last of these is a physical optics effect. The others stem from geometric optics and can be readily understood in terms of a simple camera consisting of a lens surrounded by an aperture and another aperture in front of the film which determines the size of the picture.



The camera shown above forms the image of a remote object. The illuminance E of the image on the film equals the amount of light (luminous flux) which reaches the image divided by the area of the image. Clearly the amount of light that can get into the camera is proportional to the area of the aperture around the lens which is proportional to the square of the radius of that aperture, p, or in equation form

$$\text{light flux entering camera} \propto \text{aperture area} \propto p^2 .$$

The area of the image is proportional to the square of the height of the image which is proportional to the lens focal length or in equation form

$$\text{image area} \propto h'^2 \propto f'^2 .$$

Dividing these last two proportionalities we get

$$\text{image illuminance} = \text{flux/area} \propto (p/f')^2 \propto (1/A)^2,$$

Where A is the f/number of the lens defined by

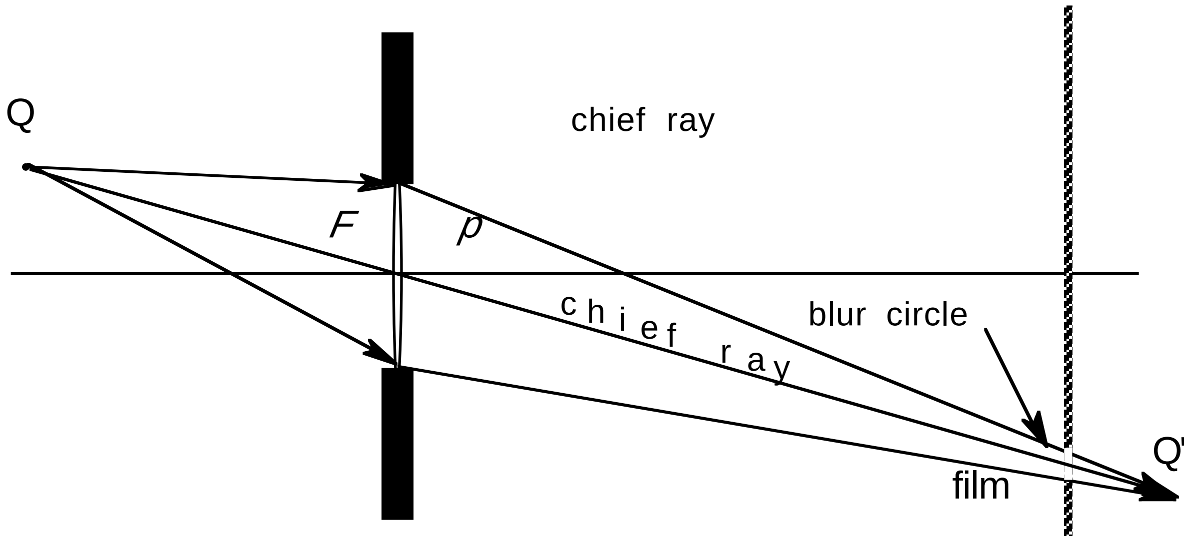
$$\text{f/number} = \text{focal length} \div \text{aperture diameter}.$$

The f/number is of great significance in photography. Photographic lenses have variable apertures with click stops set for f/numbers in the ratio of

√2=1.414… starting with 0.5 so the sequence is

f/0.5, f/0.7, f/1.0, f/1.4, f/2.0, f/2.8, f/4, f/5.6, f/8, f/11, f/16, f/22…

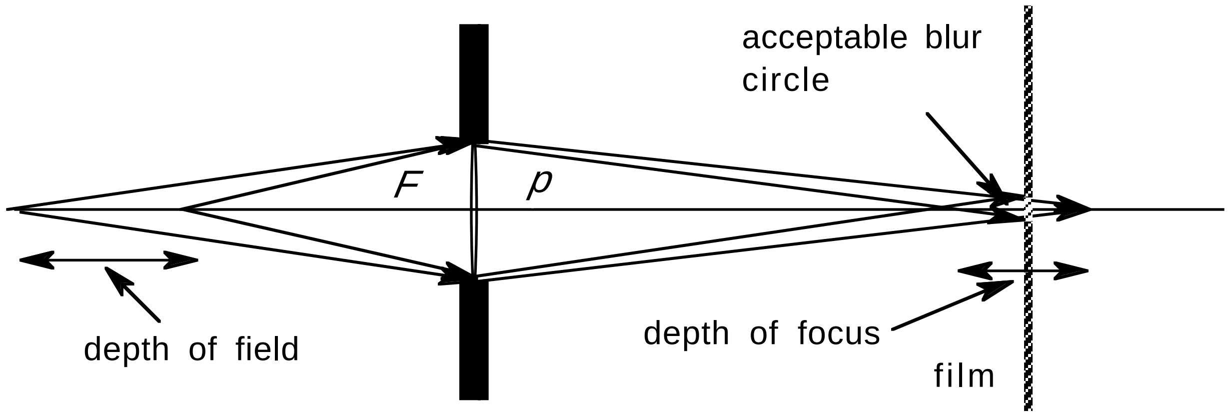
Each successive setting decreases image illuminance by a factor of 2.



The diagram above shows how the aperture determines the degree of blur of an out-of-focus image. Clearly, the blur circle radius is proportional to the aperture radius. In any optical system there is an acceptable degree of blur. Blur circles below a certain size are recognized as points. The distance through which an object can move without producing an image that is unacceptably blurred is called <u>depth of field</u>.

The ray the goes through the center of the lens aperture intersects the center of the blur circle. It is called the <u>chief ray</u> and its direction characterizes the ray bundle from B that traverses the system.

The diagram below shows the origin of depth of field.

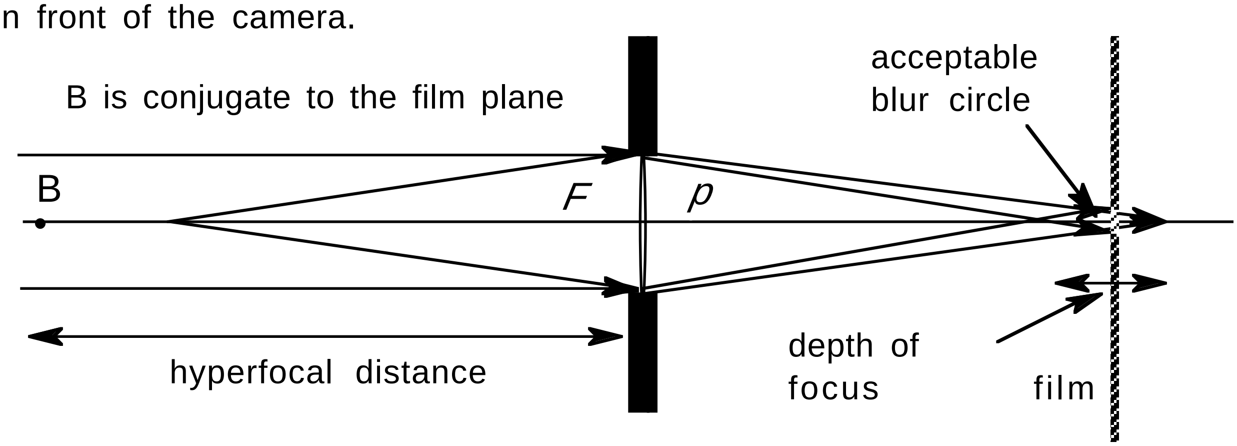


The distance through which a point source may be moved without its image exceeding the acceptable blur circle in the film plane is called the depth of field. The conjugate distance around the film plane is called the depth of focus.

If a remote point source produces a blur circle of just acceptable radius b', then an object conjugate to the film plane is said to be at the <u>hyperfocal distance</u> of the camera. The hyperfocal distance is related to the aperture radius p, the focal length f', and b' through the equation

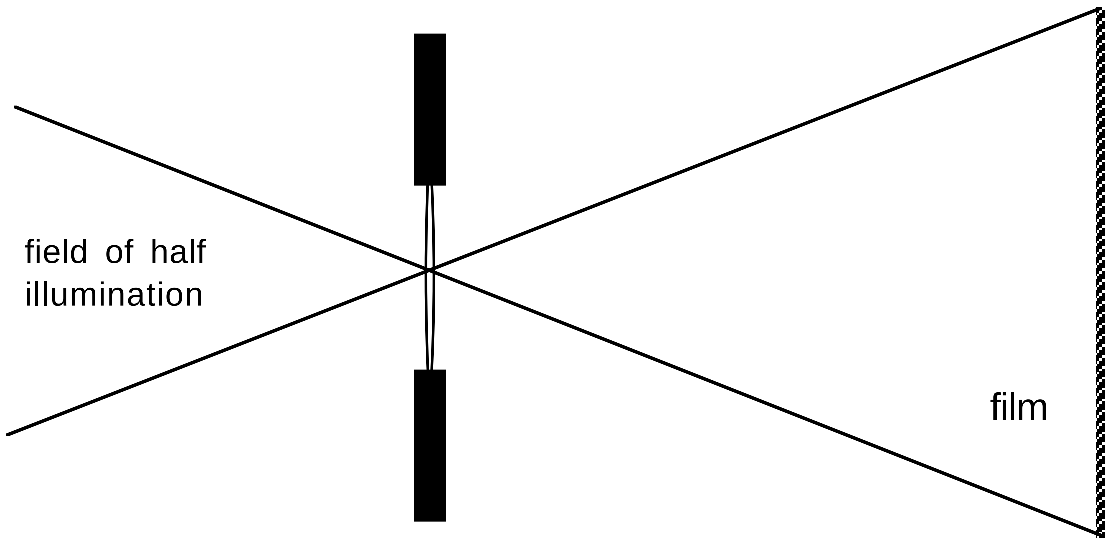
$$\text{hyperfocal distance} = -pf'/b'.$$

The minus sign is just there to remind us that the hyperfocal distance is in front of the camera.



When a camera is focused for the hyperfocal distance, its linear depth of field is infinite. Simple fixed focus cameras are set for the hyperfocal distance. The hyperfocal distance for the eye is about six meters (20 ft) which is why optometrist's eye charts are 6 meters from patients in a typical examination room.

Finally, the aperture and the size of film determine the field of view of the camera. There are several ways to define field of view. The most common is the angular subtense of the film at the lens aperture.



This is called the field of half illumination because the illumination of an image at the edge of the film is half what it would be if it were formed at the center of the film.

Because it determines the field of view of the system the edge of the film in this case is called the field stop of the camera.

