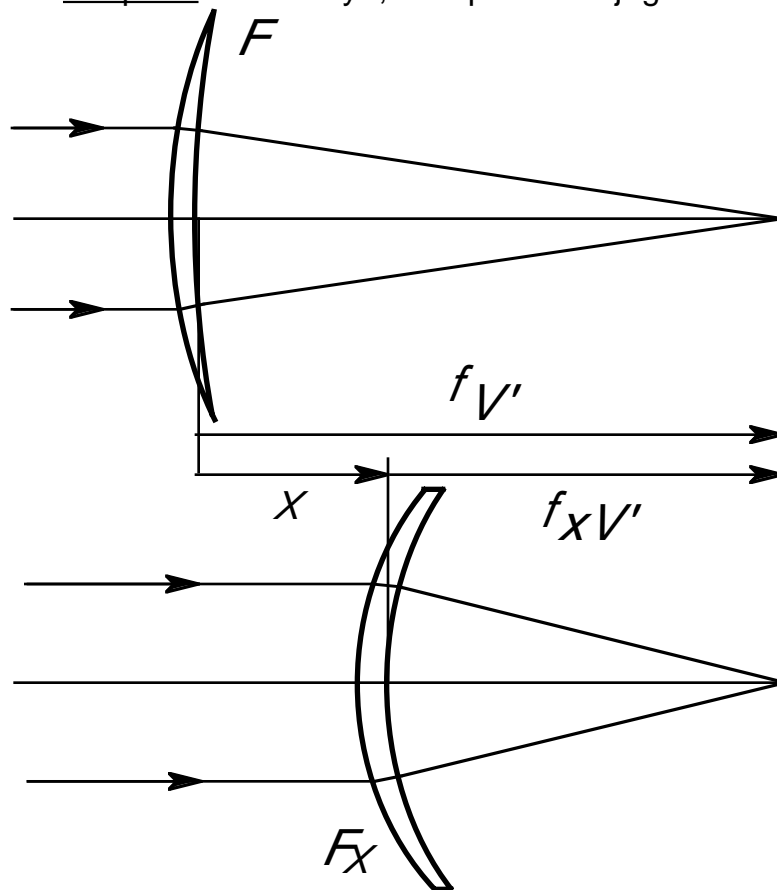


LENS EFFECTIVITY

Distance Correction

Generally it doesn't matter much where a spectacle lens sits in front of a patient's eye. In high ametropias, however, the exact lens position **can** be important. This is due to the effectivity of the lens.

Two lenses are said to have the same effective power if they form a real or virtual image of an object at the same point. In ophthalmic optics that point is usually the far point of the eye, the point conjugate to the retina of the patient.



The figure above shows two equally effective lenses of powers F and F_x . The latter lens has its back vertex a distance x behind the first lens, thus the two back vertex focal lengths are related by $f_{xV'} = f_{V'} - x$. Taking the reciprocal of this equation we obtain

$$F_{xV'} = F_V (1 - xF_V).$$

where $F_{xV'} = 1/f_{xV'}$ and $F_V = 1/f_{V'}$ are the back vertex powers of the two lenses. For astigmatic lenses, this must be applied to each meridian.

Example: What is the effective power of a +4.00D lens at a point 15cm to the right of its back vertex?

Solution: $F_x = +4.00 / [1 - (0.15)(+4.00)] = +10.00\text{D}$. This means that if we wanted to place a lens 15cm to the right of the +4.00D lens which would have the same image-forming properties with respect to collimated light as the +4.00D lens, we would have to employ a +10.00D lens. The two lenses would not work identically for sources at any other distance, however. Nor would they produce images of the same magnification.

Effectivity and Spherical Ophthalmic Lenses

In clinical practice vertex corrections may be ignored unless a correcting lens has very high power. To get some feeling for what a "high" power is, consider what happens when a lens is moved from its proper vertex distance d . As a lens is moved away from the patient's eye, to the left in our diagrams, the focal point of the lens also moves to the left, making the lens behave as though it had greater positive power or lesser negative power. Conversely, a lens moved toward the eye has greater negative power or lesser positive power. Let's calculate the movement Δx necessary to produce a given change in effective power ΔF . Use the binomial theorem to rewrite the effectivity formula when $F_V' \Delta x$ is small as

$$F_x = F_V' (1 - \Delta x F_V') \cong F_V' + \Delta x (F_V')^2.$$

If the change in effective power in moving the lens is $\Delta F = F_x - F_V'$, then this becomes

$$\Delta F \cong \Delta x (F_V')^2,$$

or,

$$\Delta x \cong \Delta F / (F_V')^2.$$

This shows that small movements of the lens become very important as F_V' increases. In clinical work it is generally assumed that power variations of 0.125D to 0.25D are significant. Below are tabulated the amount of movement permissible to keep lens power within $\pm 0.125\text{D}$ of the prescription power for various lenses:

$F_v(D)$	$d(mm)$
± 2.00	31.3
± 4.00	7.8
± 6.00	3.5
± 8.00	2.0
± 10.00	1.3
± 12.00	0.9

While low power lenses can be placed anywhere within reason, exact positioning of the correction becomes critical for powers much above eight diopters. Refractionists and dispensers must take special care with such high powered corrections. Some strategies for obtaining the proper optical correction with these high ametropias are as follows, more or less in decreasing order of advisability:



Refract the patient with trial clips over his current glasses. Measure the end result in a lensometer or combine it using an oblique cross cylinder calculation, preferably done by computer. Keep the same frame for the new glasses.



Refract using the frame of the patient's choice with trial clips over the frame. Measure the result with a lensometer.



Refract using a phoropter, but measure vertex distances with a distometer. Less accurate measurements can be made with a PD rule.



Persuade the patient to get a frame with fully adjustable nose pads.



Over-minus young patients to give them some slack if the lenses slip down their noses.



Avoid bifocals in extreme high prescriptions if the patient is willing to slide the frame down his nose to read.



Talk the patient into contact lenses. They don't slide down his nose and there are no vertex distance problems if a trial fit is done with lenses near the patient's power.

Fortunately for the optometrist (and unfortunately for the patient), patients with extreme high ametropias also tend to have poor best corrected visual acuities and relatively low sensitivities to blur, thus

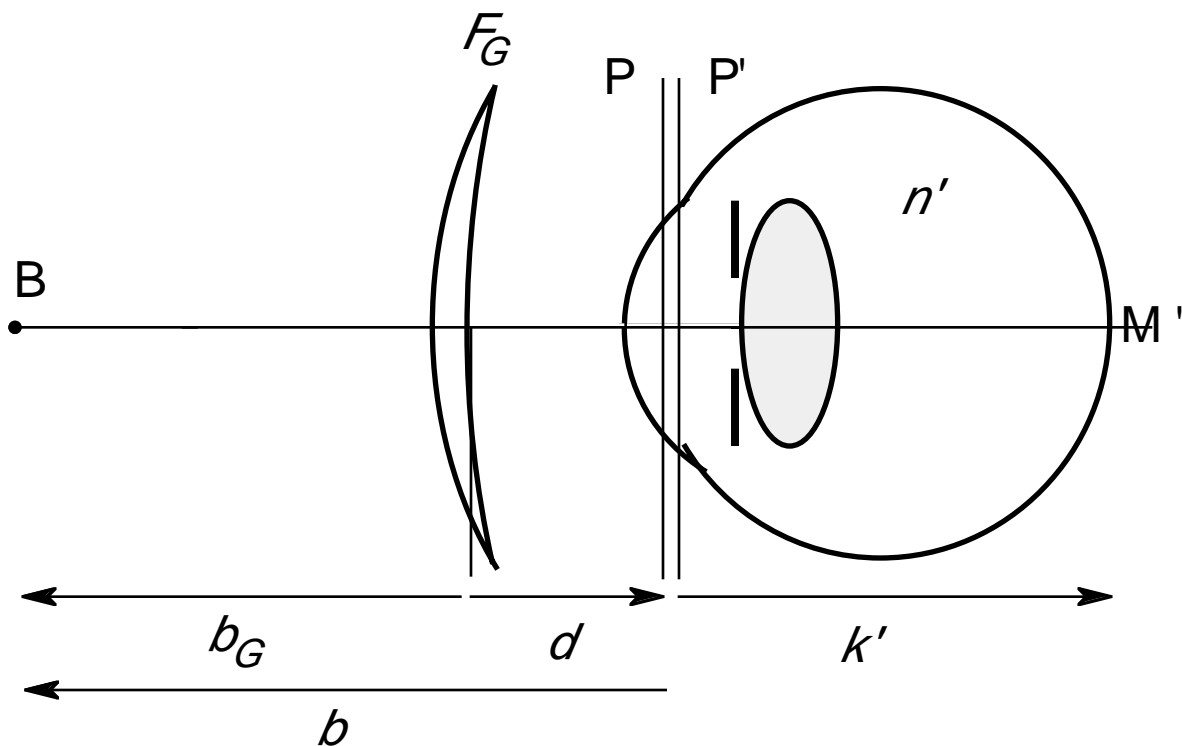
making vertex distance less critical.

Vertex correction is quite important in contact lens work when the refraction done through the phoropter has to be "vertexed down" to give the correct contact lens prescription. Vertex corrections also make the astigmatic correction slightly different in the spectacle and corneal plane although the magnitude of the difference is usually insignificant. For myopes there is somewhat less astigmatism in the corneal plane and for hyperopes somewhat more.

Near Point Correction

Vertex effects have some effect on the amount of accommodation required with a visual correction. As we'll see, this is only important in converting from glasses to contact lenses, and even then the effect is small.

Clinically, accommodation is usually measured from the spectacle plane. The spectacle plane accommodation, A_G , is simply the reciprocal of the distance from the spectacle plane to the object of regard. But the spectacle plane accommodation is not the best measure of the actual effort involved in accommodation. More significant is the ocular accommodation, A , the change in the power of the eye necessary to focus a near object. Because of effectivity, $A \neq A_G$.



Calculate principal point accommodation with reference to the diagram above. For the purposes of this calculation, assume that the spectacle lens is thin enough that its principal planes coincide and that the positions of principal planes of the accommodated eye are negligibly different from those of the unaccommodated eye. Let the object of regard and the spectacle plane be distances b and d from the first principal plane of the eye, respectively, using the sign convention of the figure. If b_G is the distance from the from the spectacle plane to the object of regard,

$$b_G = b + d. \tag{1}$$

The vergence of the light entering the spectacle lens is $B_G = 1/b_G$ so from the fundamental paraxial equation, the vergence of light leaving the lens is

$$B_G' = F_G + B_G \tag{2}$$

The vergence of light reaching the first principal plane of the eye is

$$B_P = B_G' / (1 - dB_G'). \tag{3}$$

The principal plane accommodation necessary to see the object clearly is the difference between the principal plane refraction and the vergence of light reaching the first principal plane, or

$$A = K - B_P \tag{4}$$

The ocular refraction is related to the spectacle refraction by

$$K = F_G(1 - dF_G) \tag{5}$$

Example: Calculate the accommodative response necessary in the principal plane of the eye for an object 33cm from the spectacle plane in the case of an emmetrope, an eight diopter myope, and an eight diopter hyperope. Assume spectacles are worn 15mm from the cornea and the principle point of the eye is 2mm behind the cornea.

Solution: The emmetrope accommodates $1/b = 1/(0.33\text{m} + 0.017\text{m}) = +2.88\text{D}$.

For the eight diopter myope, the vergence leaving the spectacle lens is $B_G = -8.00\text{D} + 1/(-0.33\text{m}) = -11.00\text{D}$. The vergence of light reaching the first principal plane of the eye is $B_P = -11\text{D}/[1 - (0.017\text{m})(-11\text{D})] = -9.267\text{D}$. The ocular refraction (principal point refraction) of the eye is $K = -8\text{D}/[1 - (0.017\text{m})(-8\text{D})] = -7.042\text{D}$. The required principal point accommodation is $A = K - B_P = -7.042\text{D} - (-9.267\text{D}) = +2.22\text{D}$.

For the eight diopter hyperope, the vergence leaving the spectacle lens is $B_G = +8.00\text{D} + 1/(-0.33\text{m}) = +5.00\text{D}$. The vergence of light reaching the first principal plane of the eye is $B_P = +5\text{D}/[1 - (0.017\text{m})(+5\text{D})] = +5.464\text{D}$. The ocular refraction (principal point refraction) of the eye is $K = +8\text{D}/[1 - (0.017\text{m})(+8\text{D})] = +9.260\text{D}$. The required principal point accommodation is $A = K - B_P = +9.260\text{D} - (+5.464\text{D}) = +3.79\text{D}$.

Summarizing, here is the principal plane accommodation necessary for each patient to focus a stimulus 33cm from the spectacle plane. Note how much harder the hyperope has to work.

Hyperope	+3.80D
Emmetrope	+2.88D
Myope	+2.23D

As the correcting lens moves toward the principal plane, the eye-spectacle system behaves more and more like that of an emmetropic eye, making the myope work harder and the hyperope work less. Thus a myopic patient corrected with contact lenses will have to accommodate more to focus an object a given distance from the eye than a patient wearing spectacles. A hyperope wearing contacts will have to accommodate less.

Near Cylinder Correction

Using the theory of the previous section, the astigmatic correction for near objects is somewhat different than that for remote objects.

Example: How much will a patient with prescription is $+3.00-3.00 \times 180$ have to accommodate in the principal plane to see an object 25cm from his spectacle plane? Assume the distance from the back of the lens to the cornea is 15mm and from the cornea to the principal plane of the eye 2mm.

Solution: The power cross of the the lens shows $+3.00D$ in the horizontal meridian and $0.00D$ in the vertical meridian. Work the problem through for each meridian.

In the vertical meridian $F=0.00D$ and the patient accommodates $1/b=1/(0.25m+0.017m)=+3.745D$.

For the horizontal meridian, $F=+3.00$, so the vergence leaving the spectacle lens is $B_G = +3.00D + 1/(-0.25m) = -1.00D$. The vergence of light reaching the first principal plane of the eye is $B_P = -1D/[1-(0.017m)(-1D)] = -0.983D$. The ocular refraction of the eye in this meridian is $K = +3D/[1-(0.017m)(+3D)] = +3.1610D$. The required principal point accommodation is $A = K - B_P = +3.161D - (-0.983D) = +4.14D$.

The total accommodation required is thus $+3.75 @ 090 / +4.14 @ 180 = +4.14 - 0.39 \times 180$. There is, then a $0.39D$ difference in accommodation required in the two meridians. Assuming spherical accommodation in the principal plane, to form a perfectly sharp image at near the prescription should incorporate an additional -0.40×090 over the distance prescription.

The change in apparent astigmatism at near implies an astigmatic principal plane accommodation which doesn't exist. Similarly, equal ocular accommodation in anisometric patients could produce apparently unequal amplitudes of accommodation when accommodation is measured in the spectacle plane.

Does this mean that we should prescribe an additional -0.39×090 to the above patient in reading glasses? *NO!!!* Give the patient $+4.00D$ over his distance prescription so he would not have to accommodate at all for an object $25cm$ from the spectacle plane. Pretend that we have a real $+4.00D$ lens in front of the patients $+3.00-3.00 \times 180$ distance prescription. As shown in the diagram, the $+3.00-3.00 \times 180$ lens sees collimated light-- just as it would from a remote object. But $+3.00-3.00 \times 180$ is the correct prescription for this patient for collimated light, so a $+7.00-3.00 \times 180$ lens will perfectly focus light from an object at $25cm$ on the retina.

