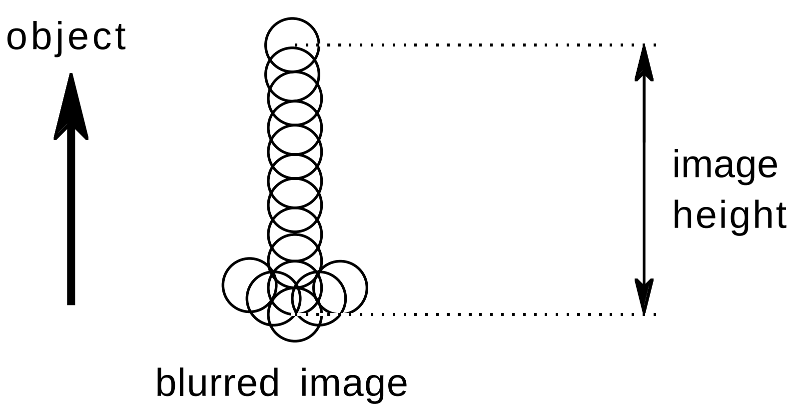
SPECTACLE MAGNIFICATION

Spectacle lenses are prescribed to focus retinal images, but they also change the sizes of those images. Clinical consequences of magnification changes range from transient spatial distortions to diplopia and amblyopia.

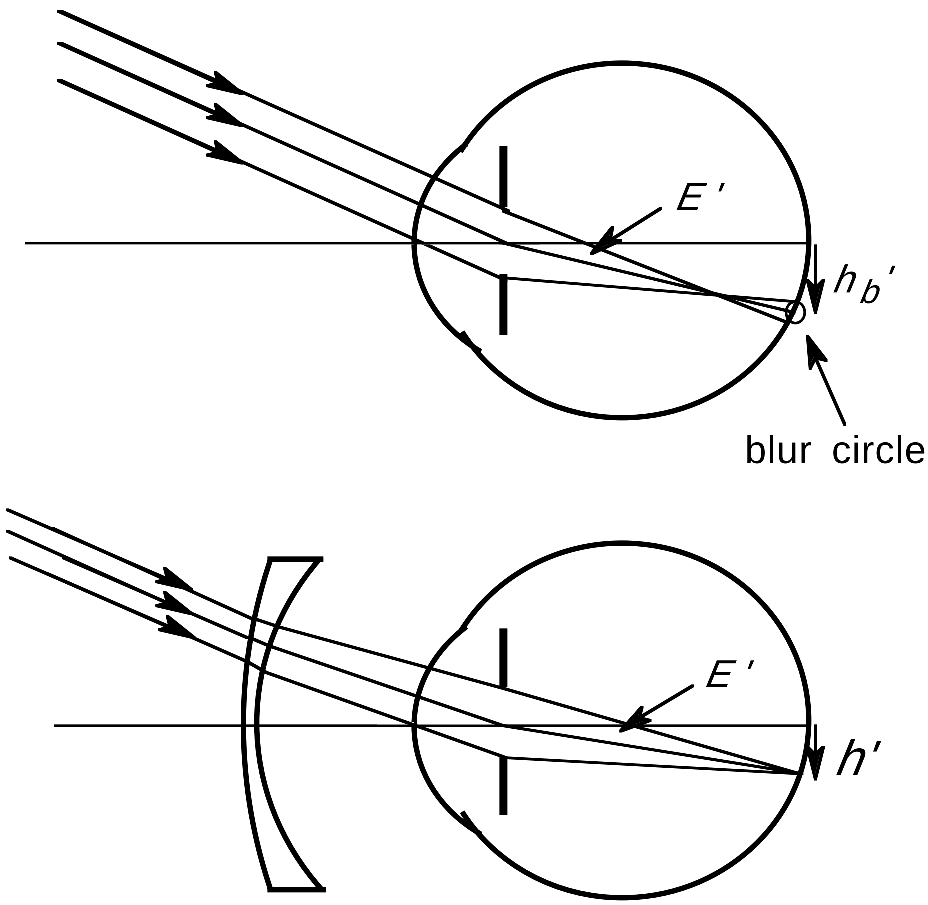
The amount of magnification produced by a visual correction is characterized by its spectacle magnification defined as

$$SM = \frac{\text{retinal image size of a remote object in the corrected eye}}{\text{retinal image size of a remote object in the uncorrected eye}}$$

First we have to understand what we mean by the image size in an uncorrected eye. Properly speaking, an uncorrected eye doesn't form an image at the retina, at least not a crisply focused one. We must, therefore, decide how to measure the height of a blurred image. By convention, that is measured from the middle of the blur circle corresponding to the top of the object to the middle of the blur circle corresponding to the bottom of the object.

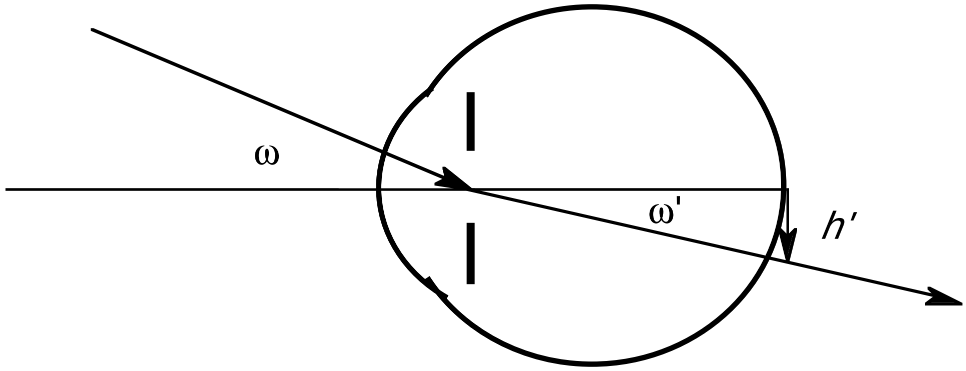


Since the chief ray from a point on the object goes through the center of the blur circle, this is equivalent to saying we measure the distance between the chief rays going to the bottom and the top of the image.



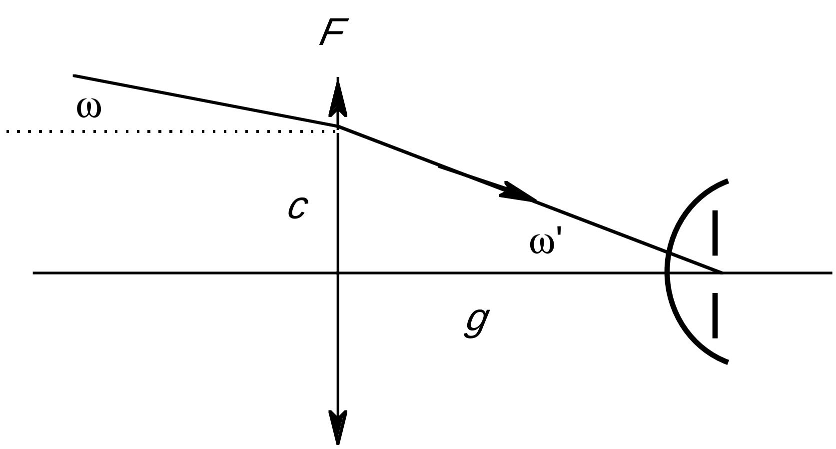
The diagrams above show a myopic eye imaging a remote object. The spectacle magnification in this case would just be SM=h'/h_b.

Offhand, it would seem impossible to calculate spectacle magnification without knowing the detailed optics of the eye. But since spectacle magnification is the ratio of image sizes formed by the **same** eye, details of the eye's optics turn out not to be necessary. The key fact here is that the angle the chief ray makes entering the entrance pupil is proportional to the angle the chief ray makes leaving the exit pupil of the eye which in turn is proportional to the height of the retinal image.



In terms of the diagram above, this says that $h' \propto \omega' \propto \omega$. So if we want the ratio of two image heights, all we need calculate is the ratio of the angles the chief ray makes with the optical axis after leaving the lens (or other optical device) placed before the eye to the angle it would have made without the lens. That, it turns out, is a property of the lens independent of the eye.

The diagram shows the chief ray from a remote object which is incident on a thin spectacle lens at height c from which it travels to the center of the entrance pupil. Let's calculate the spectacle magnification produced by the lens.



From Prentice's rule, the deviation δ of the rays is $\delta = Fc$. But $\delta = \omega' - \omega$ so $\omega' - \omega = cF$. The spectacle magnification is $\mathrm{SM} = \omega'/\omega$, so using this to eliminate ω, $\omega' - \omega'/\mathrm{SM} = Fc$. But from the diagram, $\omega' = c/g$, so $(1 - 1/\mathrm{SM})(c/g) = Fc$ and eliminating the c's and solving gives $\mathrm{SM} = 1/(1 - gF)$, the spectacle magnification of a thin lens.

The complete formula for magnification due to a thick lens can be written

$$SM=\omega'/\omega=\{1/[1-gF_V']\}\{1/[1-(t/n)F_1]\} \tag{1}$$

where

F_V' = back vertex power of the lens
F_1 = front surface power of the lens
t = thickness of the lens
n = index of the lens
g = distance from the back of the lens to the entrance pupil

The second term in curly brackets is called the <u>power factor</u> because it depends only on the back vertex power of the lens, not its shape. It is the factor we obtained for a thin lens. The first term in curly brackets is called the <u>shape factor</u> because it depends on the form of the lens through its front surface power F_1.

If $|gF_V'|$ and $|(t/n)F_1|<<1$, we can expand (1) using the binomial theorem in the form

$$(1-x)^{-1}\cong 1+x+\ \dots$$

so that

$$SM\cong 1+(t/n)F_1+gF_V'.$$

If we define percentage spectacle magnification as %SM=(SM-1)(100%) we may write

$$\%SM\cong(100\%)[(t/n)F_1+gF_V'].$$

Note that all these formulas are independent of the actual parameters of the eye other than the position of the entrance pupil, just as promised.

Example: Find the spectacle magnification of a lens with +8.00D back surface power, a +12.00D front surface surve and thickness 4.5mm if the lens is made of plastic of index 1.5 and has its back vertex 25mm from the entrance pupil of the eye.

Solution: Substituting in equation (1)

$$SM = \{1/[1-(0.025m)(+8D)\}\{1/[1-(0.0045m/1.5)(+12D)]\}$$
$$=(1.25)(1.037)=1.30$$

This lens has a 1.25=25% power factor and a 1.037=3.7% shape factor. If we'd used the approximate formula we would have obtained

$$SM \cong 1+(0.025m)(+8D)+(0.0045m/1.5)(+12D)=1+0.16+0.036=1.196.$$

The approximate formula produces significant error in the power factor here because of the high power of the lens.

The spectacle magnification of a lens may be changed somewhat by altering the various parameters. The ways of changing it may be deduced from the equation $SM \cong 1+(t/n)F_1+gF_V'$. They are as follows:

To increase spectacle magnification:

☞ Increase F_1, the base curve power

☞ Increase t, the lens thickness

☞ Decrease n, the index of the lens material

☞ Increase g for hyperopes by moving the lens away from the face

☞ Decrease g for myopes by moving the lens toward the face

As a practical matter, spectacle magnification is useful primarily as a way of comparing the magnifications of two different lenses or two meridians of the same lens.

Aniseikonia is a condition in which the patient suffers from differences in size between the images in the two eyes. This will commonly be manifested as spatial distortions due to fusing two differently shaped images of the same object. It is detected clinically by measuring those spatial distortions. The existence of aniseikonia and its attendant clinical manifestations is controversial except in the case of unilateral aphakia.

Meridional Magnification in Spherocylinder Lenses

Consider a lens with prescription Hx090/Vx180 so that it has power H in the horizontal meridian and V in the vertical meridian. Let the lens have front surface powers H_1 and V_1 in the horizontal and vertical meridians. Let's calculate the difference in spectacle magnification between the two meridians.

$$SM_{090}-SM_{180}=[1+(t/n)V_1+gV]-[1+(t/n)H_1+gH]=(t/n)(V_1-H_1)+h(V-H).$$

For lenses in minus cylinder form, the first term drops out. For plus cylinder form, both terms have the same sign and they reinforce. Thus there is less relative meridional magnification in minus cylinder form, the form used in virtually all lenses nowadays. Another way of looking at this is that the farther the astigmatic correction is from the eye, the greater the difference in spectacle magnification in the two meridians. If the correction is placed **at** the eye--as a contact lens--the minimum physically possible difference is obtained. This is why contact lens wearers with astigmatism complain so much about distortion in spectacles.

Size Lenses

Size lenses are lenses with zero back vertex power. Their only effect is to change the size of retinal images. They are used mainly as diagnostic lenses in analyzing aniseikonia.

For a size lens, the power factor vanishes, so spectacle magnification is

$$SM=1/[1-(t/n)F_1].$$

Back vertex power is $0=F_V'=F_2+F_1/[1-(t/n)F_1]$ and solving this for lens thickness gives

$$t=n(F_1+F_2)/F_1F_2=n/F_1+n/F_2=f_1'+f_2'.$$

Thus the thickness of the lens equals the sum of the secondary focal lengths of the two lens surfaces. Substituting for *t/n* in the expression for spectacle magnification of a size lens gives, after some simplification

$$SM=-F_2/F_1.$$

So the magnification of a size lens is just the ratio of the ocular surface power to the front surface power. But we've encountered these last two equations before--in the discussion of telescopes. And in fact a size lens may be considered a very low power Galilean telescope.

