RADIANT and LUMINOUS FLUX

A physicist studying a system radiating electromagnetic energy would use instruments that measure that energy in the usual MKS units. The energy emitted by the system, U, would be given in joules. The time rate of production of radiant energy, radiant flux P, would be defined as

$$P = \mathrm{d}U/\mathrm{d}t$$

where t is time. The MKS unit of radiant flux would be, therefore, joules per second or watts.

Generally the radiant flux from a source differs for different wavelengths. The flux at a given wavelength λ and in a spectral interval $\mathrm{d}\lambda$ is given by $P_\lambda \mathrm{d}\lambda$. A plot of P_λ versus λ gives the power spectrum of the radiator. Power spectra may be either continuous or discrete. The usual tungsten filament lamp has a continous power spectrum like that of figure 1. The mercury lamp spectrum shown in figure 2 is a typical line spectrum, the spectral lines corresponding to energy level transitions. Fluorescent lamps have discrete spectra superimposed on a continuous spectra. The total radiant flux P emitted by a light source is the integral of $P_\lambda \mathrm{d}\lambda$ over all wavelengths,

$$P = \int_{-\infty}^{+\infty} P_\lambda \mathrm{d}\lambda.$$

This is just the area under the P_λ curve.

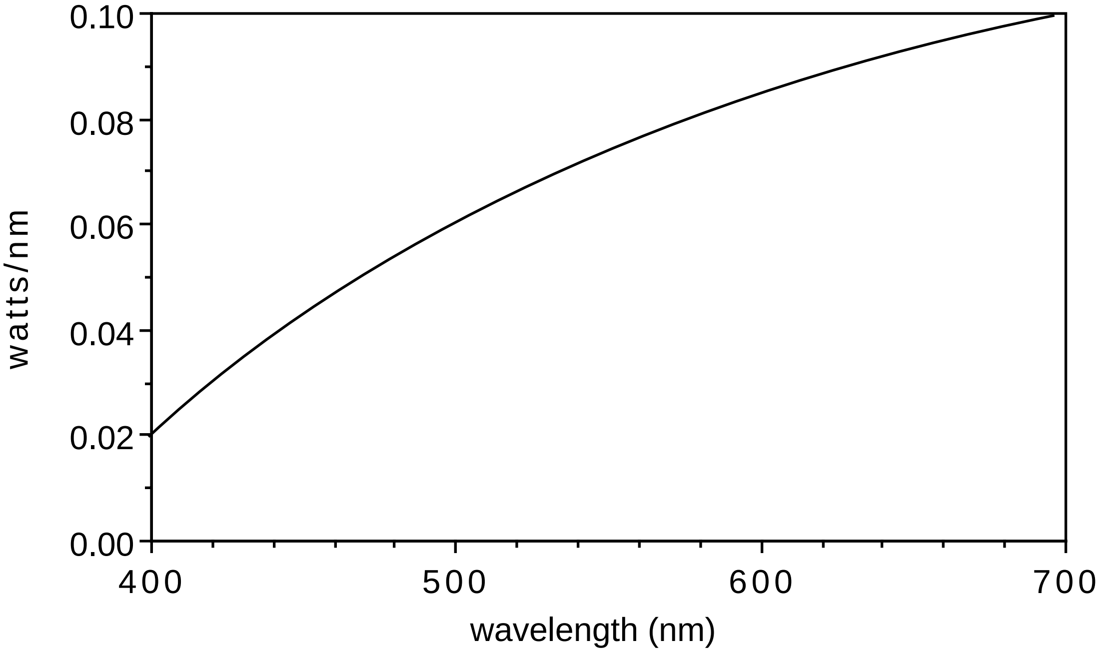

Figure 1. A typical continuous power spectrum.

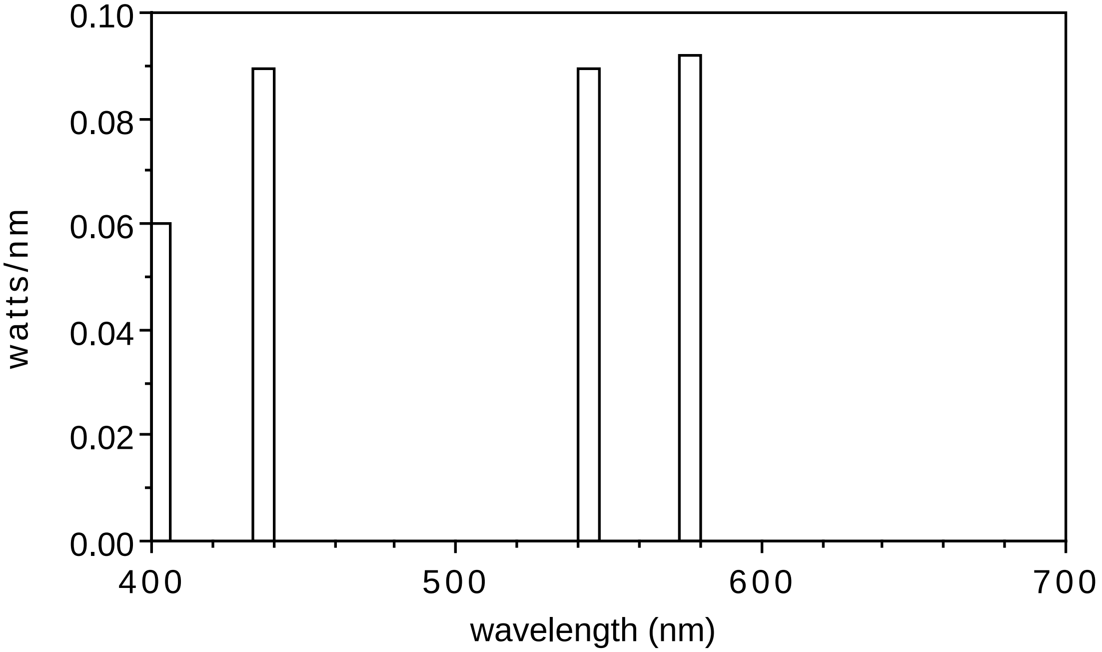

Figure 2. A typical line spectrum, that of a mercury vapor lamp.

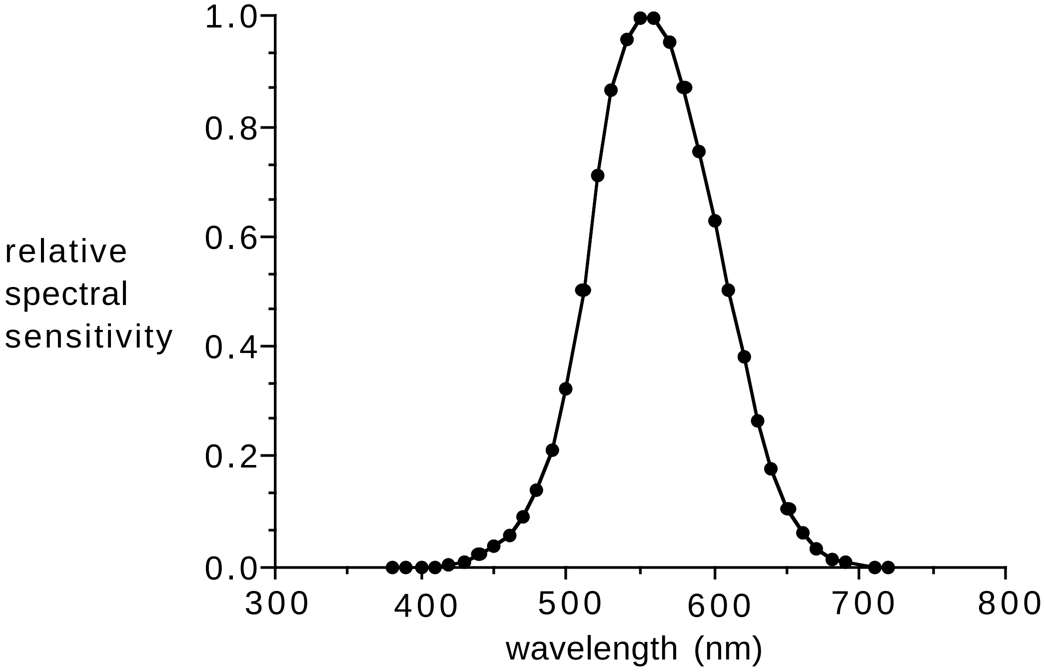

Figure 3. The relative spectral luminosity curve, v_λ.

Because visible light occupies only a small part of the electromagnetic spectrum and because the sensitivity of the eye varies considerably with the actual wavelength of the visible light, the units quantifying light energy in vision science are different from those used in physics. These units are based on the experimentally measured spectral sensitivity of the eye quantified by the spectral sensitivity function V_λ, or the relative spectral sensitivity function v_λ. One way to determine v_λ, in principle at least, would be to have a typical observer compare the amount of energy required to make the light sensation or "brightness" of light at one wavelength match that of light at another wavelength. If, for example, three times as much radiant energy were required at 500 nm as at 540 nm to produce the same magnitude of visual sensation then

$$v_{540} / v_{500} = 3.$$

(In practice the color differences in the two stimuli confound this type of experiment and more elaborate techniques are necessary. The exact results vary from observer to observer and with the state of light adaptation of the observer, but we'll ignore those effects here. The v_λ curve accepted as standard is actually an average over a great many normal observers.)

The relative spectral sensitivity function is given by the graph of figure 3 or the numerical tabulation of Table 1. The spectral sensitivity function V_λ is proportional to it according to the equation,

$$V_\lambda = (683 \text{ lumens/watt}) v_\lambda.$$

The value of the constant has been re-established from time to time so some texts may use 680 while others use 685. The value 683 is the currently accepted value.

The lumen (abbreviated lm) in the previous equation is a unit defined to characterize luminous flux in the same way the watt characterizes radiant energy. The luminous flux F_λ at wavelength λ in a range $d\lambda$ is related to the radiant flux in that interval by

$$F_\lambda d\lambda = P_\lambda V_\lambda d\lambda.$$

The total luminous flux F is obtained by integrating the above equation to obtain

$$F = \int_{410}^{720} P_\lambda V_\lambda d\lambda. \tag{1}$$

The integral is carried out in the range from 410 nm to 720 nm since that is the non-vanishing range of v_λ.

In practice the integral in equation (1) is always performed numerically using Simpson's rule. This means that (1) is approximated by the sum

$$F = (\Delta\lambda) \sum_{\lambda=410}^{720} P_\lambda V_\lambda.$$

Usually $\Delta\lambda$ is taken to be 10 nm.

Values of P_λ and V_λ can be taken from a table like table 1 or interpolated from a graph like figures 1-3.

λ(nm)	ν_λ	λ(nm)	ν_λ	λ(nm)	ν_λ	λ(nm)	ν_λ
400	0.000	500	0.323	600	0.631	700	0.004
410	0.001	510	0.503	610	0.503	710	0.002
420	0.004	520	0.710	620	0.381	720	0.001
430	0.012	530	0.862	630	0.265	730	0.000
440	0.023	540	0.954	640	0.175		
450	0.038	550	0.995	650	0.107		
460	0.060	560	0.995	660	0.061		
470	0.091	570	0.952	670	0.032		
480	0.139	580	0.870	680	0.017		
490	0.208	590	0.757	690	0.008		

table 1. A tabulation of the values of ν_λ.