PHOTOMETRY of IMAGES

Almost all optical devices form real images, on photographic film, on an array of electronic receptors, on the retina. It is frequently necessary in practical problems to determine the illuminance of that real image.

ILLUMINANCE OF THE IMAGE FORMED BY A THIN LENS

To simplify the discussion let's consider image formation by a simple camera such as that in figure 1. The camera has a thin lens of focal length f' and radius p. The rim of the lens limits the amount of light which may enter the system. The camera forms a real image of the Lambert radiator of area S'. The object is a distance l from the entrance pupil and the image a distance l' from the exit pupil. The object is a Lambert radiator of luminance L. We want to find the illuminance, E', of the image formed by the lens.

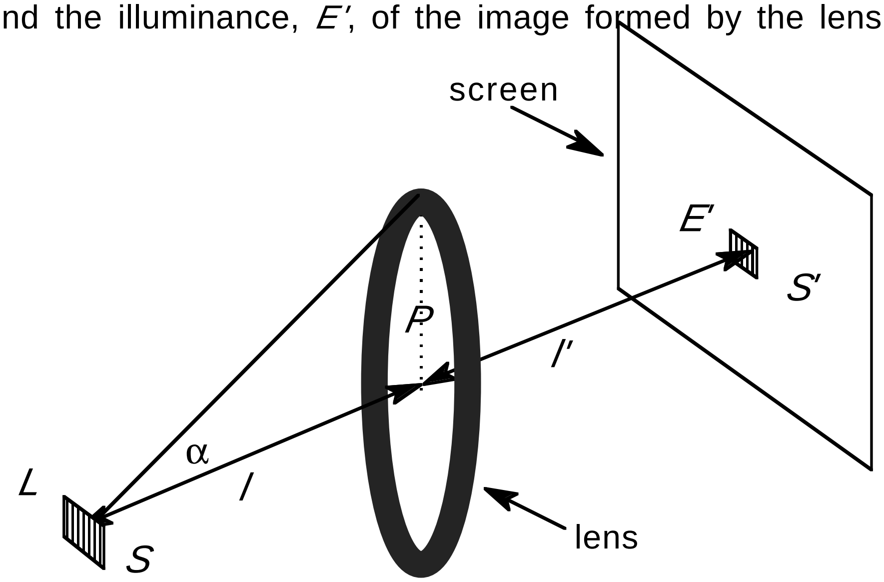

Figure 1. The image of an object of area S and luminance L formed by a thin lens of radius p has area S' and illuminance E'.

The first step is to find out how much flux gets into the optical system. That is just the flux that is admitted by the rim of the lens. The intensity of the source area in a direction θ is $I(\theta)=LS\cos\theta$. The flux $dF(\theta)$ from S into a solid angle $d\omega$ in a direction θ is given by

$$dF(\theta)=LS\cos\theta d\omega,$$

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where θ specifies the orientation of the solid angle $d\omega$.

To find the total flux F into the entrance pupil, we must integrate over the area of the lens,

$$F = \int_{lens} dF(\theta) = \int_{lens} LS\cos\theta d\omega = LS\int_{lens}\cos\theta d\omega. \tag{1}$$

In the paraxial limit where the object distance is much greater than the radius of the lens, the angle θ is small enough that $\cos\theta \cong 1$ over the whole lens. Since the solid angle subtended by a small disk at a point on its axis is $\omega=\pi\alpha^2$, if the lens subtends half angle α, then the integral in (1) becomes

$$F=\pi L\alpha^2 S\ .$$

If τ is the fraction of light transmitted by the system, the luminous flux leaving the exit pupil is

$$F'=\tau F=\pi\tau L\alpha^2 S\ .$$

If the area of the image is S', the illuminance of the image is just

$$E'=F'/S'=\pi\tau L\alpha^2(S/S')\ .$$

If the linear magnification of the image is m, $S'/S=m^2$ (why?), and the illuminance of the image becomes

$$E'=\pi\tau L\alpha^2/m^2. \tag{4}$$

Though derived for the special case of a thin lens in air, equation (4) actually holds in the paraxial limit for *any* optical system if α is reinterpreted as the half-angular subtense of the entrance pupil. If the system is a wide aperture system then the equation holds with the replacement $\alpha\Rightarrow\sin\alpha$.

From the fundamental paraxial equation of geometric optics, the magnification of the image can be written,

$$m=l'/l. \tag{5}$$

From figure 1 and equation (5),

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$$|\alpha|=p/|l|=|m|(p/l').$$

Substituting into equation (4),

$$E'=\pi\tau L(p/l')^2 \tag{6}$$

In the calculation above we've assumed that the object surface is parallel to the lens so that the normal to the object surface is parallel to the optic axis. What would happen if the object in figure 1 were tilted so that its normal made an angle γ with the optic axis? Here in equation (1) $\cos\theta \Rightarrow \cos\gamma$ and the flux entering the exit pupil would be $F=\pi L\alpha^2 S\cos\gamma$. Likewise, the image size would become $S' \Rightarrow \cos\gamma S'$. The two factors of $\cos\gamma$ would cancel out in numerator and denominator, leaving the image illuminance the same no matter what the angle of observation of the object.

Even in optical systems more complicated than the thin lens camera, image illuminance is proportional to object luminance. Since the amount of visual sensation depends on retinal illuminance, which is proportional to object luminance, luminance is the appropriate quantity to characterize the amount of light coming from a visual stimulus. Since the retinal image illuminance is independent of the angle at which the object is viewed, the perception of brightness of a Lambert radiator will be the same no matter what the angle of observation.

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CAMERA LENSES and f/NUMBERS

For a remote object, $l' \simeq f'$, where f' is the focal length of the lens and from (6) the illuminance of the photographic image becomes

$$E' \simeq \pi\tau L(\rho/f')^2$$

or

$$E' = \frac{\pi\tau L}{4(\text{f/number})^2} \qquad (7)$$

where the f/number is defined as the ratio of focal length to entrance pupil diameter. Equation (7), then, shows that image illuminance for a remote object goes as the inverse square of the f/number. Equation (7) is adequate for most realistic cases since object distances are much greater than camera focal lengths. Equation (4) is necessary in close-up photography. From (6) we note that the ratio of the illuminance, $E'(l)$, of an object at a finite distance l to $E'(\infty)$, the illuminance of a remote object, is

$$E'(l)/E'(\infty) = (f'/l')^2,$$

hence (7) still holds if we make the replacement

$$\text{f/number} \Rightarrow (\text{f/number})(l'/f')$$

The factor (l'/f') is called the <u>bellows extension factor</u> and is used in calculating exposures in close-up photography.

THE TELECONVERTER

The <u>teleconverter</u> or <u>telextender</u> is a diverging optical system which may be mounted between the camera lens and camera body to increase the magnification of the photograph. The principal of the teleconverter is illustrated in figure 2.

Teleconverters are labelled by the magnification they produce. The most common is the 2X extender which doubles image size. Since the entrance pupil of the lens is unchanged, the telextender spreads light flux over a larger area and decreases image illuminance.

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The effective f/number with a teleconverter in place is the actual number multiplied by the tele-converter magnification.

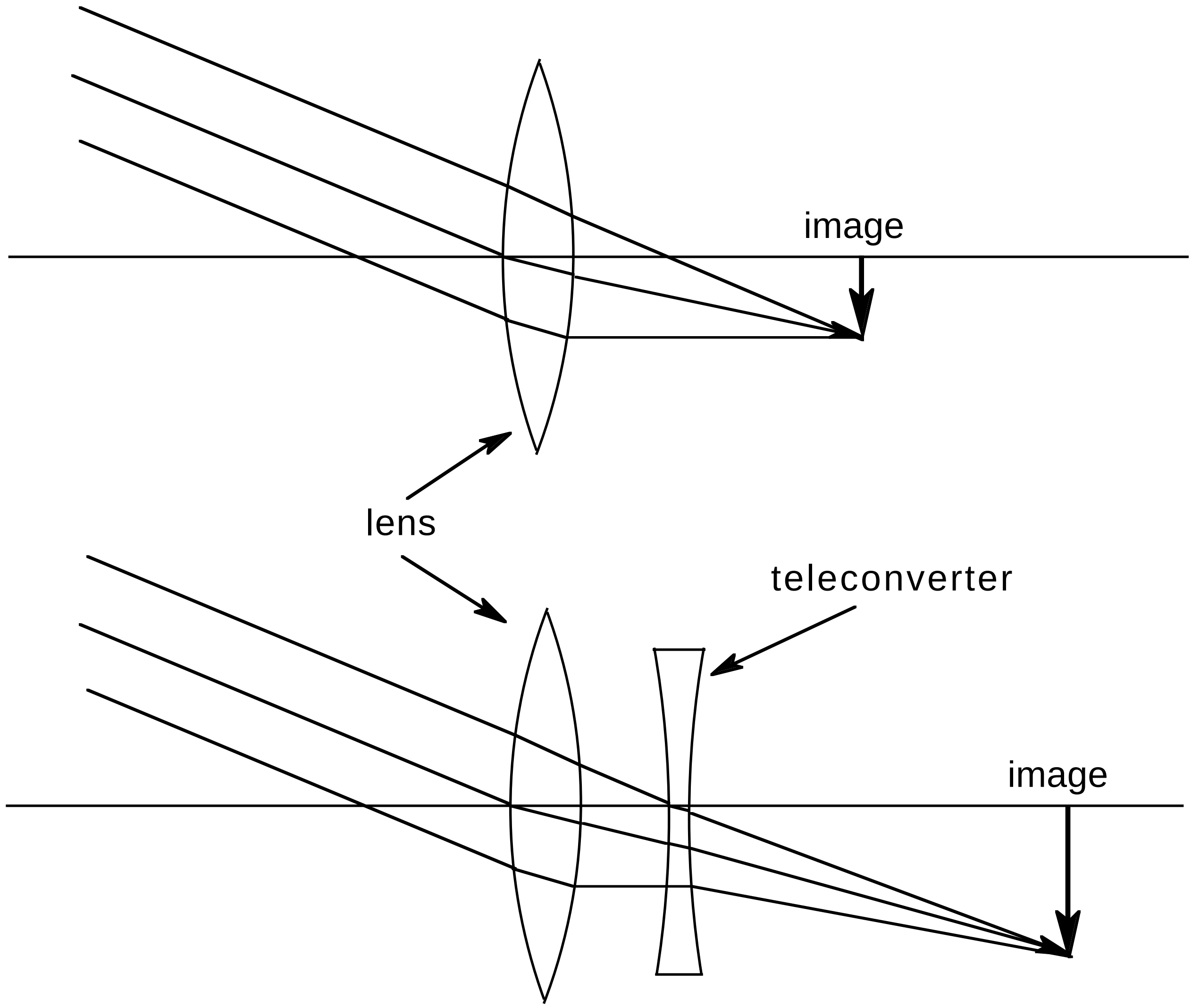

Figure 2. Optics of the teleconverter.

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THE PHOTOGRAPHIC ENLARGER

Another interesting application of (4) is that of the photographic enlarger. We can think of an enlarger as consisting of simply an object (the photographic negative) imaged by a thin lens with entrance and exit pupils of the same size, onto a screen (the photographic paper), the whole thing being in air. In this case (4) becomes

$$E'=\pi\tau L(p/l')^2, \qquad (8)$$

Eliminating l using the fundamental paraxial equation $1/l+1/f'=1/l'$,

$$l'=(1-m)f'=(1+|m|)f'.$$

Substituting this into (8) gives

$$E'=\pi\tau L(p/f')^2/(1+|m|)^2=\pi\tau L/[4(\text{f/number})^2(1+|m|)^2]$$

Photographers doing darkroom work can use the fact that $E'\propto[1/(1+|m|)]^2$ to calculate exposures when an enlargement size is changed.

THE LUMINANCE METER

Equations (4) and (7) are the basis for the construction of luminance meters. Such meters consist of optical systems with known parameters such as f/number, τ, etc. The illuminance, E', of the image focused by the system is measured and from it object luminance is inferred. Thus even luminance meters actually operate by measuring illuminance.

Modern cameras have built-in light meters which guide manual exposure calculations or set the shutter and/or aperture automatically to give best estimated exposure. These meters are, in fact, luminance meters.

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COSINE-FOURTH FALL-OFF LAW

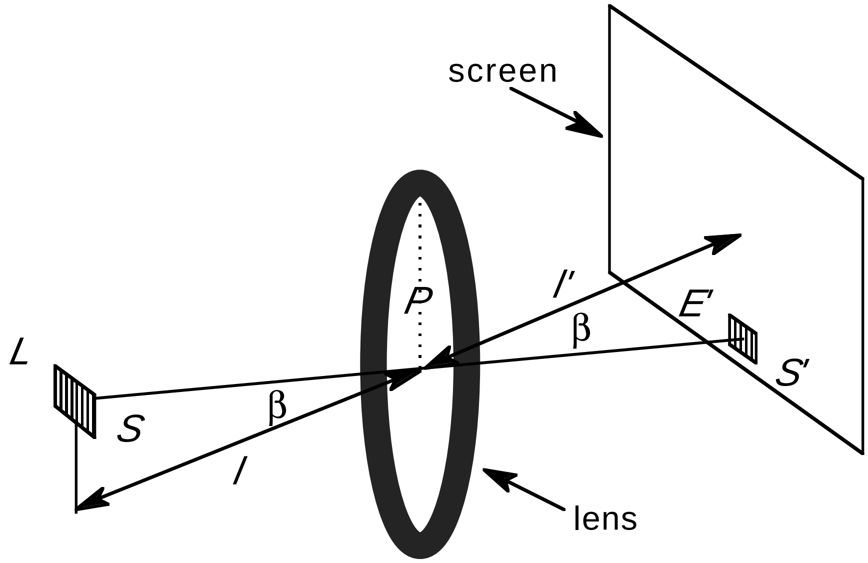

Figure 3. Illuminance of an off-axis image.

Let's find the illuminance of the image if the object were moved off the optic axis as shown in figure 3. Again we must evaluate equation (1). If the angle between the optic axis and the line connecting the object and the center of the entrance pupil is β, then over the area of the entrance pupil $\cos\theta \cong \cos\beta$ so (1) becomes

$$F = \int_{lens} dF(\theta) = \int_{lens} LS\cos\theta d\omega = LS\cos\beta \int_{lens} d\omega. \tag{9}$$

But $d\omega$, the angular subtense of the entrance pupil, is given by $d\omega = \cos\phi dA/r^2$, where dA is an increment of area of the entrance pupil, r is the distance from the source to dA and ϕ specifies the orientation of dA. From geometry $\phi \cong \beta$ over the entrance pupil and $r = |l|/\cos\beta$, so the integral in the preceding equation becomes

$$\int_{lens} d\omega = \int_{lens} \frac{\cos\phi}{r^2} dA = \frac{(\cos\beta)^3}{l^2} \int_{lens} dA.$$

Thus the flux entering the entrance pupil is

$$F = LS(\pi p^2)\cos^4\beta/l^2.$$

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The illuminance of the image is then given by

$$E'=F'/S'=\tau F/S'=\pi\tau B(S/S')(p/l)^2\cos^4\beta=[\pi\tau B\alpha^2/m^2]\cos^4\beta.$$

But from (4), the term in square brackets is just the illuminance the image would have had if the object were on the optic axis. Calling that latter illuminance E'_{axial}, the preceding equation becomes

$$E'=E'_{\text{axial}}\cos^4\beta. \tag{10}$$

Equation (10) is an expression of the cosine-fourth fall-off of image illuminance at the edge of the field of view. It is significant in certain wide angle optical systems such as cameras with short focus lenses or in the eye. In most applications, however, it may be ignored.

The photometry of two particular imaging systems is especially important--the camera and the eye itself.

RECIPROCITY

Most sophisticated photographic objectives have variable apertures. Aperture settings are designated as f/numbers, an f/number of eight being written as f/8. The apertures are equipped with click-stops at standard settings. These settings lie in a geometric sequence starting at 0.5 and with a common ratio of $\sqrt{2}=1.4142\ldots$ The first several terms in this sequence are

f/0.5, f/0.7, f/1, f/1.4, f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16, …

Increasing the f/number one click along this sequence halves the film illuminance.

Over the range of most frequently used photographic settings, the density of exposure on film depends on only the total amount of light reaching the film, hence decreased film illuminance can be compensated by increasing the length of time the shutter is open. In mathematical terms, exposure is the same for different combinations of shutter and aperture if $t/(\text{f/number})^2$ is the same, where t is the duration of the exposure. This

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is called film reciprocity.

Manual cameras have shutter speed settings with click stops arranged so that each setting leaves the shutter open about half as long as the one before. The settings are usually designated by their reciprocals which are in a geometric sequence starting with one and with a ratio of two or nearly two. The terms in the sequence are

1, 2, 4, 8, 15, 30, 60, 125, 250, 500, 1000, 2000, ...

This arrangement means that when a photographer increases his f/number one click he can keep exposure constant by decreasing his shutter speed one click.

RETINAL ILLUMINANCE

We could, in principle, use an appropriate generalization of equation (4), to calculate the retinal illuminance in the eye, but that would require a detailed knowledge of the optical parameters of the eye which we usually lack. In many situations, however, psychophysical experiments, for example, it is only necessary to compare the retinal illuminance of the eye under one set of circumstances to that under another. Since retinal illuminance is proportional to the area of the pupil of the eye, retinal illuminance is often characterized by a unit called the "troland", which is defined as

$$\text{retinal illuminance (trolands)} = [\text{target luminance } (cd/m^2)][\text{entrance pupil area } (mm^2)].$$

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LUMINANCE OF AN AERIAL IMAGE

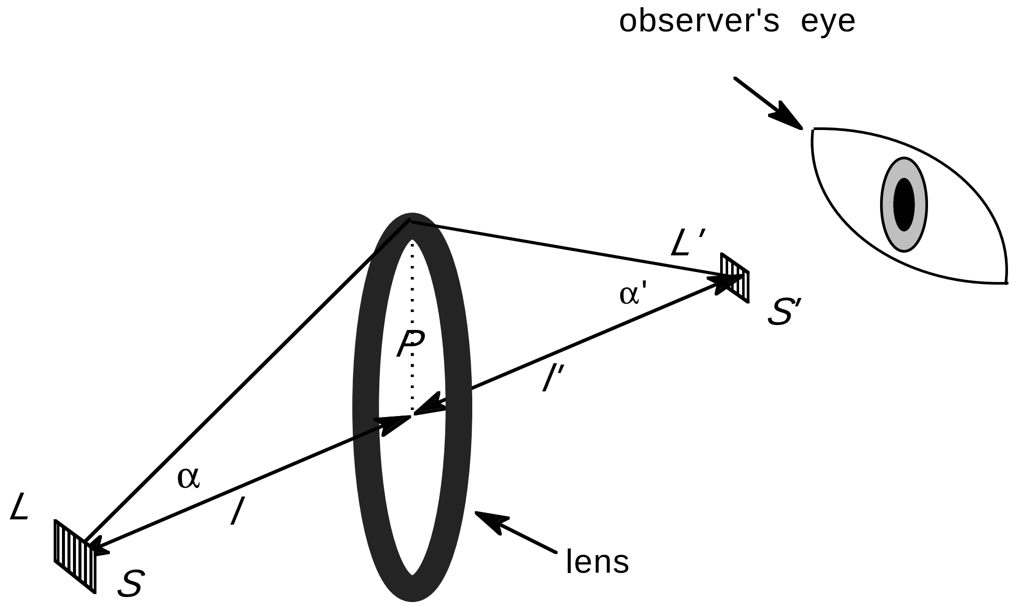

Figure 4. A lens forms an aerial image of luminance L' of an object of luminance L.

Finally, let's work out what the apparent luminance of a luminous object is when it's observed through a telescope or other optical instrument. Figure 4 shows an observer looking at the image of an area S of luminance L formed by a lens. Assume that the pupil of the eye is the aperture stop for the eye-lens system. For simplicity, assume the object area is parallel to the lens and lies on the optic axis.

As we've already shown, in the paraxial limit the flux entering the entrance pupil is

$$F=\pi L S \alpha^2,$$

and the flux leaving the exit pupil is

$$F'=\pi\tau L S \alpha^2.$$

The observer will see the image as having luminance L' where, by the definition of luminance,

$$L'=F'/(S'\omega').$$

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Here ω' is the solid angle subtended by the exit pupil at the image. We've assumed here that the subtense of the entrance pupil of the eye at the image is smaller than ω'. If α' is the half angle subtended by the lens at the image, then $\omega' \propto \pi\alpha^2$ and combining this with the preceding results gives

$$L' \doteq (\pi\tau L S\alpha^2)/(\pi S'\alpha'^2) = \tau A L (S/S')(\alpha^2/\alpha'^2).$$

Since $|\alpha| = |p/l|$ and $|\alpha'| = |p/l'|$, the preceding equation becomes

$$L' \doteq \tau L (S/S')(l'/l)^2.$$

But since $m^2 = S'/S = l/l'$ and

$$L' \doteq \tau L.$$

For untinted lenses $\tau \cong 1$ so object and image luminances are effectively the same. Since the luminance of the image of one lens is the same as the luminance of the object, it follows that the luminance of any device made from a combination of lenses will also be the same as that of the object. Hence the luminance of the image formed by a telescope or a microscope is the same as the luminance of the object seen with the naked eye.[1]

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[1]Although it's not readily apparent in this discussion, the above derivation assumes that the pupil of the eye is the aperture stop of the eye-instrument optical system.

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