

IMAGE PHOTOMETRY PROBLEMS

More difficult problems are indicated with an asterisk.

1. A primitive camera has as its objective a thin lens 4 cm in diameter with 0.1 m focal length. What is the illuminance of the image formed by this camera of a remote object of 500 lx luminance?
2. Work out the generalization of equation (6) when object and image space have indices n and n' .
- 3.* If the image of a surface of area S has area S' , show why $S'/S=m^2$ where m is the magnification of the image.
- 4.* For an enlarger we showed that image illuminance, $E \propto 1/(1+|m|)^2$. But equation (4) seems to say that $E \propto 1/m^2$. Explain this paradox.
5. A photographer is making prints of a standard 35 mm negative, a rectangle 24 mm x 36 mm. He gets a satisfactory 7.3 cm x 11 cm enlargement if he exposes the photographic paper for 20 seconds. How many seconds must he expose the paper to get a 12 cm x 18 cm enlargement? (Photographic paper obeys reciprocity.)
6. A member of the so-called "f/64" school of photography achieved great depth of field in their photographs by using the smallest apertures available on their cameras. With the large format cameras they had, that was f/64, hence the name. Suppose a light meter indicates that the best exposure on a cloudy bright day will be obtained at f/11 with a shutter of 1/125 sec. What shutter would be used at f/64?

- 7.* When a camera is used to shoot a picture of a tree, best exposure is obtained with $f/8$ @ $1/60$ second. If the photographer wants to take picture of one leaf of the tree at 1:1 magnification under the same light and with the same shutter speed, what aperture setting should he dial into his camera objective? Assume the entrance and exit pupils of the camera are about the same diameter.
- 8.* In a comparison photometer an observer measures the luminance of a source by subjectively matching its brightness to that of a calibrated source. If carefully done by a good observer, these measurements are quite accurate. Is this consistent with the statement that "luminance must *always* be inferred from an illuminance measurement?"
9. A disk shaped Lambert radiator has radius 10 mm and 1000 asb luminance.
- Plot the illuminance of a surface parallel to and on the axis of the disk for disk-to-surface distances from 50 mm to 500 mm.
 - The disk is photographed with a camera which has an $f/4$ lens of 40 mm focal length with its optic axis along the disk axis. There is negligible absorption of light by the lens. On the graph of part (a), plot the film plane illuminance for disk-to-lens distances from 50 mm to 500 mm. For simplicity treat the lens as if it were a thin lens.
 - Indicate on your graph the asymptotes of both curves for large values of distance.
 - The curves of part (a) and part (b) have a significantly different dependence on distance. What is the physical reason for that?
- 10.* A photographic objective has focal length f' and entrance and exit pupils of equal diameter. For large object-to-entrance pupil distances / equation (7) says the film plane illuminance of an image of luminance L will be $E' = (\pi L/4)/(f/\text{number})^2$. What is the range of object-to-entrance pupil distances for which this equation will be accurate to within 10%? What will this range be for a lens of 50 mm focal length?

- 11.* As an observer moves away from a light source, the illuminance of the retinal image remains the same. Does this contradict the inverse square law?

ANSWERS to SELECTED PROBLEMS

1. 62.8 lm/m²
2. $E' = \pi \tau L (\rho/l)^2 (n'/n)^2$
5. 44 seconds
6. 1/4 second. At such a slow shutter speed it would be necessary to mount the camera on a tripod. The f/64 photographers specialized in landscapes and still lives.
7. f / 4
8. Yes, it is consistent. With a comparison photometer the observer is actually matching the retinal illuminances of the unknown and calibrated sources.
9. In part (a) the image illuminance equation is plotted where $\pi L = 1000$ asb. For large values of l this becomes the inverse square law with the asymptote $E = 0$. In part (b) equation (6) is plotted. It has the asymptote (7), which in this case reduces to $E' = (\pi L/4)/(f/\text{number})^2$ where again $\pi L = 1000$ asb. Image illuminance rises to a constant as $l \rightarrow \infty$ because the lens gathers the light to focus it on an image. The further from the disk, the less light enters the lens, but that light is concentrated within an ever smaller image to keep illuminance E' constant.

10. Since film plane illuminance increases with distance l , to have an error of less than 10%, $E(l)$ must satisfy the inequality $E(l)/E(\infty) > 0.9$. But $E(l)/E(\infty) = (f/l)^2 \Rightarrow l' < f/\sqrt{0.9} = 1.0541 f$. From the fundamental paraxial equation, $1/l = 1/f + 1/l' \Rightarrow l = f l' / (f - l')$ so $l > [1.0541 f^2] / [(1 - 1.0541) f]$ and finally, $l > 19.49 f$, the desired relationship. Applying this to a 50 mm lens gives $l > 19.49 \times 50 \text{ mm} = 0.974 \text{ m} \approx 1 \text{ meter}$. Since most photography is done with the subject well beyond 1 meter from the camera, $E = (\pi \tau L) / [4(f/\text{number})^2]$ is sufficiently accurate for practical purposes, especially since film exposure latitude is usually significantly greater than 10%.
11. The illuminance of the observer's face does follow the inverse square law. So does the illuminance of the observer's pupil, but the change in the luminous flux reaching the retina is exactly compensated by the change in magnification of the retinal image, so the retinal illuminance stays the same. (This assumes that the object is observed from a distance much greater than the focal length of the eye.) So the inverse square law does not apply when light is refocused as is the case with light reaching the retina, and there is no contradiction.