

EXTENDED LIGHT SOURCES and LUMINANCE

DEFINITION OF LUMINANCE

The point source approximation of the previous section is only valid when the dimensions of the source are much smaller than the distance of the observer from the source. More generally, light sources have extended areas. Extended sources can be primary sources which generate light energy or secondary sources which reflect light from a primary source.

To generalize from point to extended sources, break up the extended sources into infinitesimal luminous areas dS . These areas are point sources of light of intensity dI . A straightforward way of characterizing an extended source would be in terms of the intensity per unit area, dI/dS . But that's not what is actually done. Instead, the source is characterized by a quantity called luminance, symbolized by L , defined by

$$L = dI / (\text{projected area of } dS).$$

As shown in figure 1, the projected area is just the apparent area of a source subtended along the line of sight of an observer. (We encountered projected area already in the discussion of solid angle.) If an area dS is tilted so that its normal (the line perpendicular to the surfaces) makes an angle θ with the line of sight, its projected area is $\cos\theta dS$ and the previous equation becomes

$$L = dI / (\cos\theta dS). \tag{1}$$

Luminance characterizes the amount of light output from a finite source in much the same way intensity characterizes the light output of a point source. Luminous intensity and luminance bear much the same relation to one another in photometry as charge and surface charge density do in electrostatics.

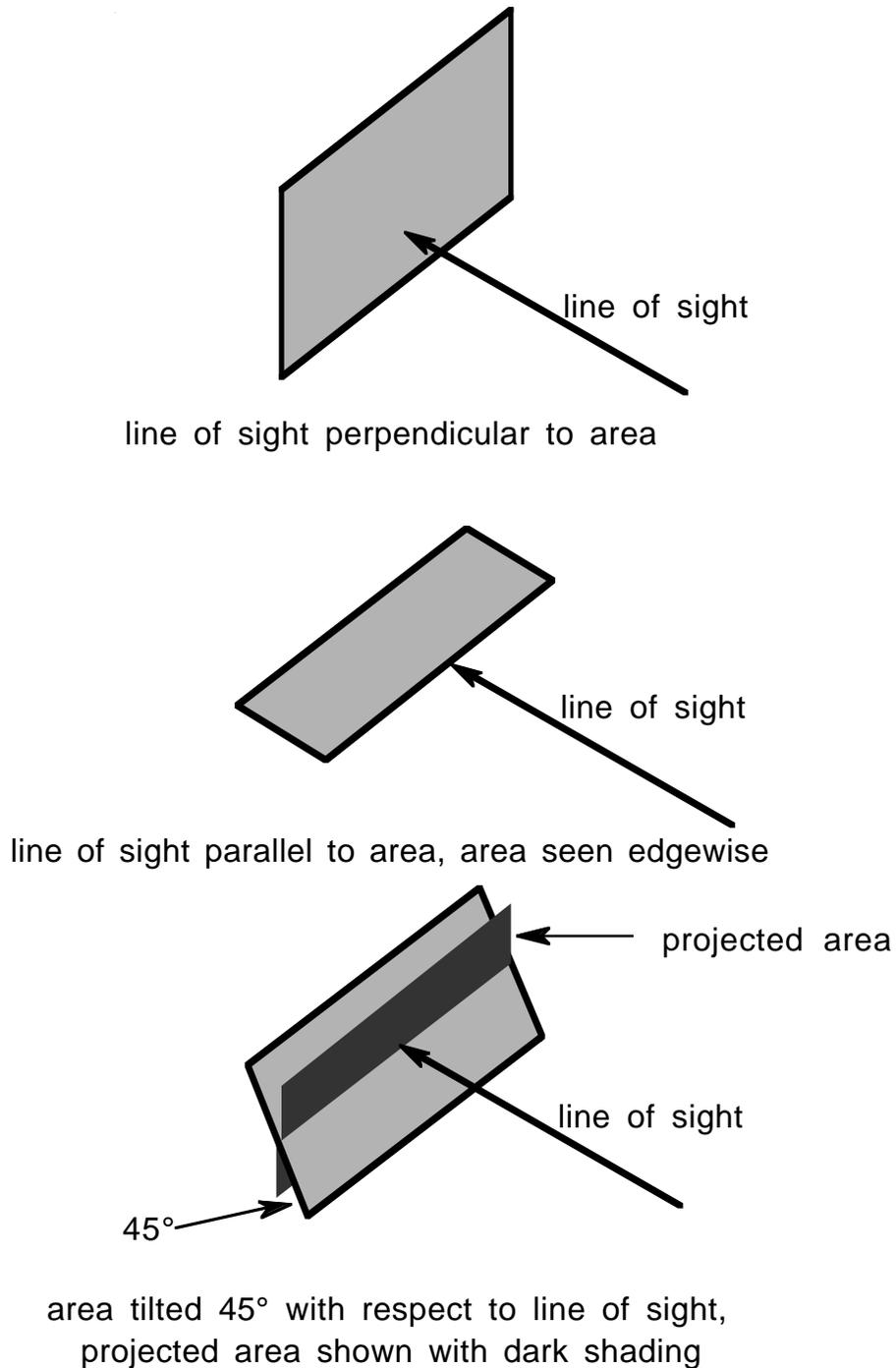


Figure 1. Projected area is the subtense of an area along the line of sight. If the line of sight is perpendicular to the area, the area and projected area are the same. If the line of sight is along the edge of the area, projected area is zero. If an area is tilted 45° with the line of sight, projected area is $\cos 45^\circ = 0.707$ times the area.

Luminance is the most mysterious entity of all those associated with photometry. The use of projected area in the definition seems especially obscure and arbitrary. But it turns out that with this definition, the retinal **illuminance** of an image is proportional to object **luminance**, as we'll see later when we discuss photometry of images. Since the retinal illuminance drives visual sensation, this explains why luminance was called brightness and symbolized by B in older literature.

LAMBERT RADIATORS

In general the intensity and luminance of a source varies with the angle of observation. The distribution of scattered luminous flux for several secondary sources is shown in figure 2. The most interesting case is that of figure 2(c), the perfect matte surface or Lambert radiator. Here the amount of scattered flux and, hence, intensity varies as $\cos\theta$, where θ is the angle between the line of observation and the normal to the surface. But from (1) this means that luminance is a constant, the same in every direction. (This is another consequence of defining luminance in terms of projected area.) Together with the discussion of the previous paragraph, this means that a matte surface will produce the same retinal illuminance and hence the same brightness, when viewed from any angle. And indeed this is the case for most common objects, including articles of clothing, walls, pieces of paper, etc. Exceptions would be shiny objects reflecting light, certain slide projection screens, and the screens of some portable computers.

From (1), the metric and English units of luminance are candelas per square meter or lux (lx) and candelas per square foot or foot-candela (ft-cd).

Returning to our original problems, combining (1) with the inverse square law of photometry we obtain for the illuminance dE of an area dA by a source of area dS a distance r away,

$$dE = L \cos\theta \cos\phi dS / r^2. \tag{2}$$

In general L depends on θ , but most of the time we'll restrict our considerations to the more tractable case of the Lambert radiator for which luminance is independent of the angle of observation.

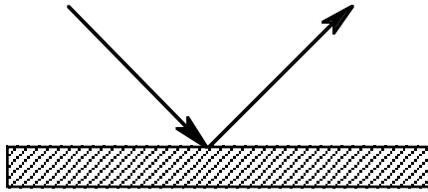


Figure 2(a). In specular reflection, like that from a mirror, the angle of incidence equals the angle of reflection.

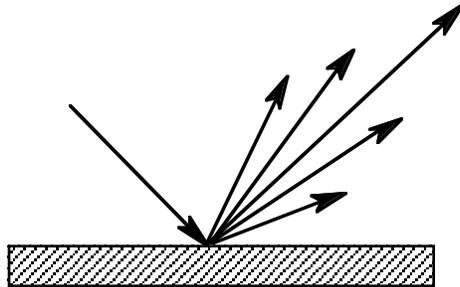


Figure 2(b). Spread reflection occurs when light strikes a surface like frosted glass. Light scatters into a bundle around the specular pencil.

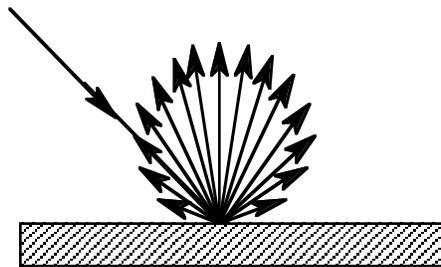


Figure 2(c). Lambert radiators such as most matte surfaces give diffuse reflection which re-radiates incident light in all directions according to the cosine law.

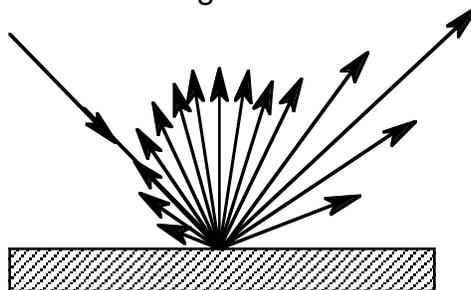


Figure 2(d) Surfaces like slick paper magazine pages will superimpose a spread and a diffuse reflection, mixed reflection.

LUMINANCE AND ILLUMINANCE

To apply (2) to realistic, finite areas we must integrate it to obtain

$$E = \int dE = \int L \cos \theta \cos \phi dS / r^2. \quad (3)$$

We'll apply (3) to two interesting cases.

The first is that of a flat or nearly flat source of linear dimensions $\ll r$. In this case θ and ϕ are effectively constant over the surfaces, as is r , so assuming we have a Lambert radiator with constant luminance L the integral becomes

$$E = \int dE = \int_S L \cos \theta \cos \phi dS / r^2$$

or just

$$E = L \cos \theta \cos \phi S / r^2 \dots \text{for a small source}$$

The second case is that of the illuminance of a surface on the axis of and parallel to a disk-shaped Lambert radiator as illustrated in figure 3. Here the radiator has luminance L and radius a and is a distance R from the receiving surface.

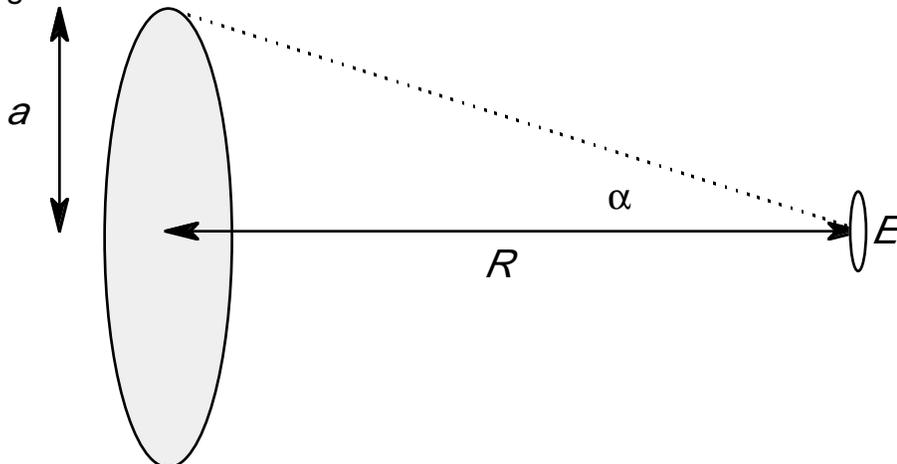


Figure 3. A disk of luminance L and radius a subtending half angle α at the center of a co-axial receiving surface.

To apply (1), first break the circle up into a series of concentric ring shaped areas as in figure 4. The inside radius of each of these areas is ρ and the outside radius $(\rho+d\rho)$ so that the area of each ring is

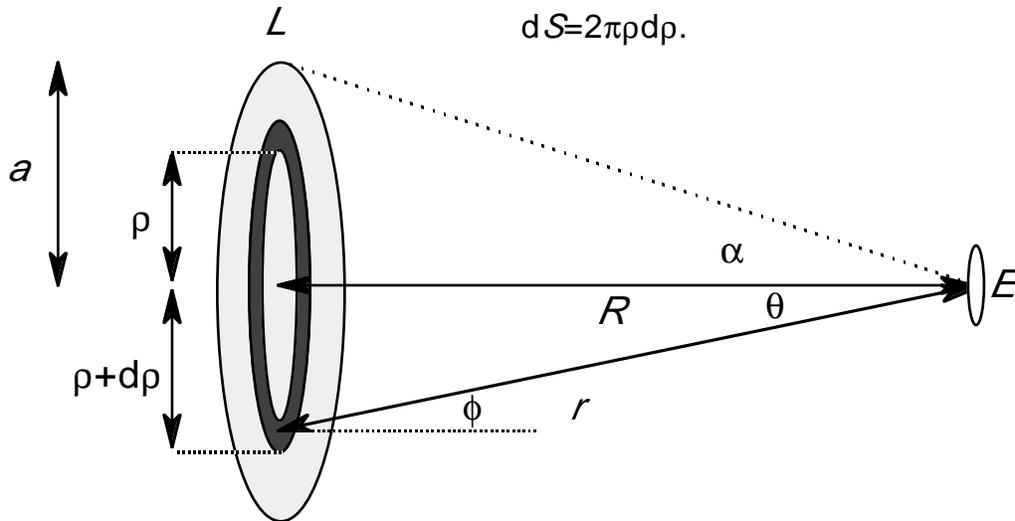


Figure 4. To calculate the illuminance due to the disk break it up into infinitesimally thin rings.

(Why?) Any point on a ring is a distance

$$r = \sqrt{R^2 + \rho^2}$$

from the center of the receiving surface and makes an angle

$$\phi = \arccos(R/r) = \theta.$$

Combining these results with (2), we obtain for the contribution of one ring,

$$E = \int_0^a \frac{2\pi LR^2 \rho d\rho}{(R^2 + \rho^2)^2}$$

Summing the contributions of each ring gives the integral

$$E = \int_0^a \frac{2\pi LR^2 \rho d\rho}{(R^2 + \rho^2)^2}$$

Evaluating the integral (how?) and simplifying, we obtain
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$$E = \frac{\pi L a^2}{R^2 + a^2}. \quad (4)$$

If we rewrite the equation in terms of α , the half angle subtended by the disk, (4) takes the simple form

$$E = \pi L \sin^2 \alpha. \quad (5)$$

It is helpful in interpreting (4) and (5) to consider two special cases:

Case 1:

The receiving plane is very far from the luminous disk so that $R \gg a$, $\sin \alpha \cong \alpha$. From (4), E becomes

$$E \cong \frac{\pi L a^2}{R^2}. \quad (6)$$

Thus from a large distance the circle looks like a point source of intensity

$$E = L S, \quad (7)$$

where $S = \pi a^2$ is the area of the disk.

Combining (6) and (7) we recover the inverse square law,

$$E \cong \frac{I}{R^2} \quad \dots \text{ for large } R. \quad (8)$$

Another form for E at large distances comes from using the small angle approximation for α to get

$$E = L \omega$$

where ω is the solid angle subtended by the source at the receiving plane.

Case 2:

The receiving plane is very close to the luminous disk so that $a \gg R$, and $\alpha \cong 90^\circ$.

From (4) or (5),

$$E \cong \pi L. \quad (9)$$

Because the receiving plane is so close to the source, the source behaves, in effect, like an infinite plane. We can say, then, that a luminance meter with its head parallel to an infinite plane will give a reading

$$\begin{aligned} E(\text{lm/m}^2) &= \pi L(\text{cd/m}^2), \\ E(\text{ft-cd}) &= \pi L(\text{cd/ft}^2). \end{aligned} \quad (10)$$

Note that the dimensions and area of the source do not appear anywhere in equation (10).

Measurement of illuminance in this case is also a measure of the luminance of the source, the two quantities differing only by a multiplicative constant. Luminance and illuminance are numerically equal if we define a new set of units, the so-called Lambert units, in which

$$\begin{aligned} \text{one apostilb} &= (1/\pi) \text{cd/m}^2, \\ \text{one foot-Lambert} &= (1/\pi) \text{cd/ft}^2. \end{aligned}$$

With these units we can make the replacement¹

$$\pi L(\text{cd/unit area}) \Rightarrow L(\text{Lambert units}).$$

We can summarize the meaning of this equation in the conversion table below:

multiply cd/ft ² by π to get foot-Lamberts
multiply cd/m ² by π to get apostilbs
divide foot-Lamberts by π to get cd/ft ²
divide apostilbs by π to get cd/m ²

Hence $1000 \text{ cd/m}^2 = 1000\pi \text{ apostilbs} = 3141 \text{ apostilbs}$,
 $200 \text{ ft-L} = 200/\pi \text{ cd/ft}^2 = 64 \text{ cd/ft}^2$, and so forth.

Now we can write for the infinite plane

$$\begin{aligned} E(\text{lm/m}^2) &= L(\text{apostilb}), \\ E(\text{ft-cd}) &= L(\text{ft-Lambert}). \end{aligned} \tag{11}$$

The apostilb is abbreviated "asb" and the foot-Lambert is abbreviated "ft-L". Though Lambert units are not used much any more, it is important to be familiar with them since they are encountered frequently in the older literature. (Unit fanciers might further enjoy knowing that one lumen per square meter is a lux, lx; one candela per square meter is a nit, abbreviated nt.)

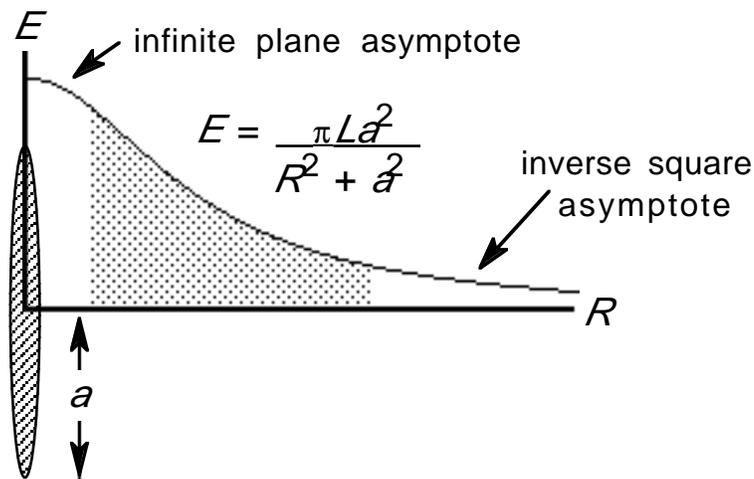


Figure 5. Illuminance, E , of a coaxial parallel plane due to a luminous disk of radius a and luminance L (indicated in the diagram) plotted as a function of the distance R separating the disk and receiving plane.

Figure 5 graphs equation (4). The exact shape of the luminous source only matters for values of R in the shaded region of the graph. Values to the right of the shaded area approach the inverse square law asymptote $E = LS/R^2$, where S is the total area of the source, and values to the left approach the constant value $E = \pi L$ associated with an infinite plane source.

it turns out, equations (7), (8), (10) and (11) are true for remote and

proximal points regardless of the shape of the uniformly luminous plane surface. The actual shape of the source is only important for intermediate distances.

LUMINANCE AND VISUAL ACUITY

As illustrated in figure 7, the visual acuity of a target like an optometric test chart depends on its luminance. As the figure shows, acuity is essentially constant for high luminance levels down to about 50 ft-L, but drops dramatically at levels below 10 ft-L. That's why a test chart should have at least 10 ft-L luminance and illuminance recommendations for reading areas are chosen to give reading matter luminances of about 50 ft-L.

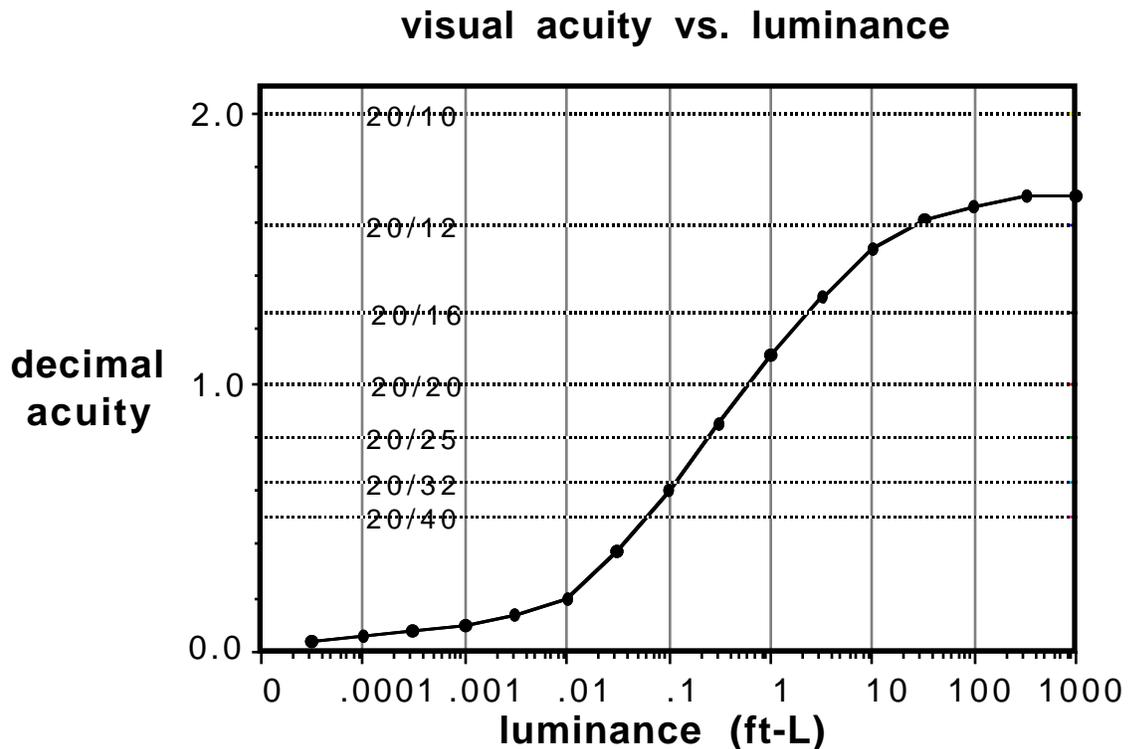


Figure 7. The dependence of visual acuity on target luminance for a typical observer

It also explains why early presbyopes often have problems only in dim light. At high luminance levels, acuity is adequate to read material held at normal distances. At low light levels, however, they can't compensate diminished visual acuity by moving the object of regard closer to their eyes because of their diminished accommodation.

¹These equations may look contradictory. To understand them better, consider a simple case of length x which can be given in feet or inches. It is true that 1 foot=12 inches, **and** $x(\text{inches})=12x(\text{feet})$. Try it with a couple of examples to prove it to yourself.