SOLID ANGLE PROBLEMS

More difficult problems are indicated with an asterisk.

1. Meteorologists measure cloud cover by estimating the percentage of the sky blocked by clouds, e.g. 100% is totally overcast while 0% is a completely blue sky. If a meteorologist on a flat, treeless desert estimates 30% cloud cover, what solid angle does the blue sky subtend?

2. A man 5 ft tall is lowered into a dry well 25 ft deep and 10 ft in diameter. What solid angle is subtended at his eye by the sky when he stands at the center of the well bottom and looks up?

3.* A patient has a circular scotoma (a blind spot) which subtends 20° (10° half angle) at the nodal point of his eye. (a) What solid angle does the scotoma subtend? (b) A Goldmann perimeter is a hemispherical bowl 33 cm in radius. The extent of a scotoma is measured clinically by finding what portion of the sphere falls within the scotoma when the patient's eye is at the center of the sphere. What area would this patient's scotoma subtend on the Goldmann perimeter?

4. Plot equations (3) and (4) for $0° \leq \alpha \leq 90°$. Within what limits is the approximate formula (3.4) accurate to within 10%? To within 1%?

5.* A tangent screen is a flat screen used to plot the extent of patient's scotomas, much like the Goldmann perimeter (see problem 3). When the screen is placed one meter from a patient suffering from quinine poisoning she shows a scotoma which is projected onto the screen as a ring shaped area concentric with her line of sight having inside diameter of 60 cm and outside diameter of 80 cm. What solid angle does the scotoma subtend? What area would the scotoma subtend on the Goldmann perimeter?

6. What is the half angle subtense of a disk that subtends one steradian of solid angle at a point?

7.* As derived in the text, the solid angle subtense of a disk of radius a at an axial point a distance R from its center is

$$\omega=2\pi R\int_0^a[\rho/(R^2+\rho^2)^{3/2}]d\rho$$

Evaluate the integral in this equation.

8. Curtis Joseph, one time goalie for the St. Louis Blues, got $1,000,000 per year to "cut off the angle" of skaters trying to shoot the puck into the goal. (He gets a lot more now!) In fact, Cujo was cutting down the solid angle subtended by the mouth of the goal at the shooter. Give three ways he could accomplish this.

9.* Working from basic geometric principles, show that the projection of a rectangular area A onto another surface is $A\cos\phi$, where ϕ is the angle between the normals of the two surfaces.

10.* The integral for the solid angle subtense of a disk at an axial point P can be written in terms of the variable r, the distance from the point to an infinitesimal ring the instead of ρ, the radius of the infinitesimal ring. Write and evaluate this integral.

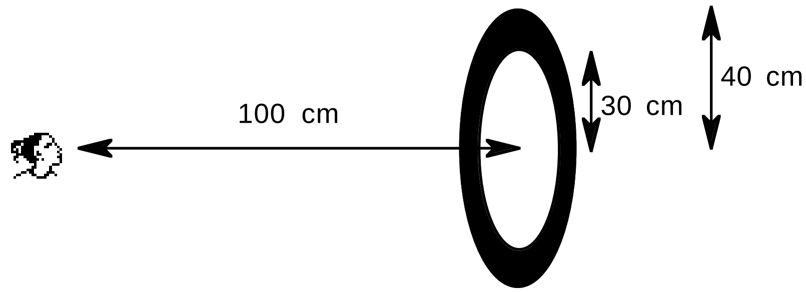
ANSWERS to SELECTED PROBLEMS

1. 4.4 str

2. 0.2 str

3. (a) 0.0955 str; (b) 104 cm^2

5. The geometry of this problem is shown below:



Find the solid angle subtense of the scotoma by subtracting the subtense of a disk of radius 30 cm from the subtense of a disk of radius 40 cm. Because the distance R=100 cm from subject to tangent screen is comparable to the radius of the scotoma, we must use the exact formula for solid angle subtended by a disk of radius a at an axial point,

$$\omega = 2\pi\left[1 - \frac{R}{\sqrt{R^2 + a^2}}\right].$$

The solid angle subtenses of outer and inner disks are, respectively,

$$\omega_2 = 2\pi\left[1 - \frac{(100\,\text{cm})}{\sqrt{(100\,\text{cm})^2 + (40\,\text{cm})^2}}\right] = 0.449\,\text{str},$$

$$\omega_1 = 2\pi\left[1 - \frac{(100\,\text{cm})}{\sqrt{(100\,\text{cm})^2 + (30\,\text{cm})^2}}\right] = 0.265\,\text{str}.$$

The subtense of the ring-shaped scotoma is just the difference between these two, ω=0.184 str.

A Goldmann perimeter is a sphere of radius 33 cm (see problem 3) at the center of which the patient places her eye. The area on the sphere subtended by this scotoma would be

$$A = \omega r^2 = (0.184\ \text{str})(33\ \text{cm})^2 = 200\ \ \text{cm}.$$

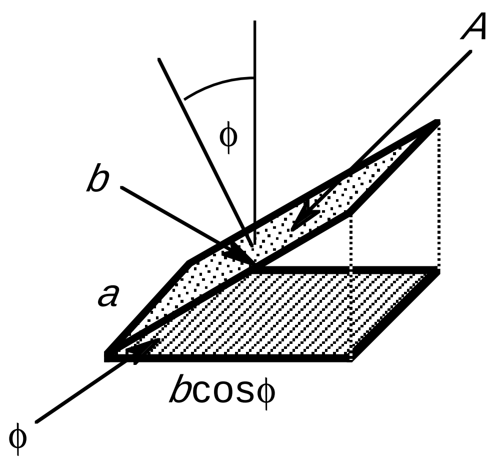
6. 33°

7. Hint: integrate using change of variables, the new variable being $u=(R^2+\rho^2)$.

8. Curtis could:

(a) turn his body toward the shooter so the normal to his body is along the line with the shooter;
(b) spread out his stick and pads to present maximum area to the shooter;
(c) come out of the net toward the shooter.

9. The geometry of the situation is shown below. We've taken A to be a rectangle of sides a and b so that $A=ab$. Clearly, the projected area has sides a and $b\cos\phi$ so the projected area equals $ab\cos\phi=A\cos\phi$.



10. The integral becomes

$$\omega = 2\pi R\int_R^{\sqrt{R^2+a^2}} \frac{dr}{r^2}.$$