DIFFRACTION BY CIRCULAR APERTURES, RESOLUTION and VISUAL ACUITY

VISUAL ACUITY

The first thing done in an optometric examination is to measure the patient's "vision", actually visual acuity. A variety of charts have been evolved for the purpose. A portion of a typical chart might be as follows:

Figure 1. A visual acuity chart.

The patient's job is to read as far down the chart as possible. The farther down he reads, the better his vision. What visual acuity tests do is to measure the eye's ability to separate two images, to tell whether there are one or two.

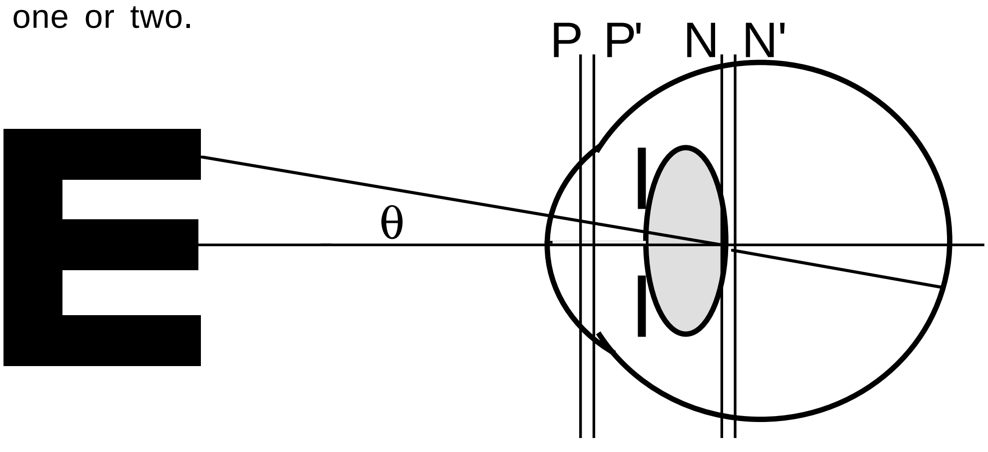


Figure 2. The angle of resolution θ is the angle between the smallest perceptible details subtended at the nodal point.

The most common system for indicating the resolving power of the eye is the Snellen system. In it letters or other targets are rated according to the distance at which a normal eye (one which can resolve 2 points of angular separation 1 minute of arc) can just identify them. A 20 ft target, for example, could be identified at 20 ft, a 50 ft target could be recognized 50 ft from the subject, and a 10 ft target would have to be brought to within 10 ft of the subject. One then exposes the subject to a series of these targets at some standard testing distance, typically 20 ft. The acuity of the patient is given by the following fraction:

$$\text{Snellen acuity} = (\text{testing distance}) \div (\text{rating of smallest detected letter})$$

An acuity of 20/15, for example, indicates that the patient saw the "15 foot" letter at a testing distance of 20 feet. The smaller the bottom number, the better the acuity. A person with 20/20 acuity can just resolve a grating in which the lines are separated by one minute of arc.

What factors contribute to the limits of the eye's ability to resolve two points or lines?

- ☞ aberrations of the eye
- ☞ defocus, refractive error
- ☞ receptor density at the retina
- ☞ light scatter within the eye
- ☞ diffraction

This last factor is intrinsic to the function of the eye or any other optical system. For the same reason the image of a thin slit is not a thin line, the image of a point source is not a point, even in a perfect optical system. The rest of the discussion below will be about the characteristics of the image of a point source in an optical system with a circular aperture stop.

DIFFRACTION BY CIRCULAR APERTURES

A perfect optical system should, according to geometric optics, produce a point image of a point object. But look what the retinal image of a point source really looks like!

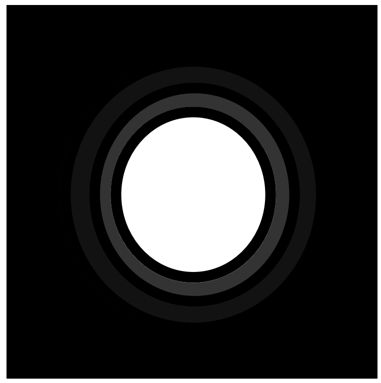


Figure 3. Diffraction pattern with a circular aperture.

Here's how that result comes about. Assume an optical system has a circular aperture. Collimated light passes through the aperture and goes to a remote screen.

To find the result at the screen, treat the aperture at the screen as a whole bunch of point sources, as per Huygen's principal (figure 4).

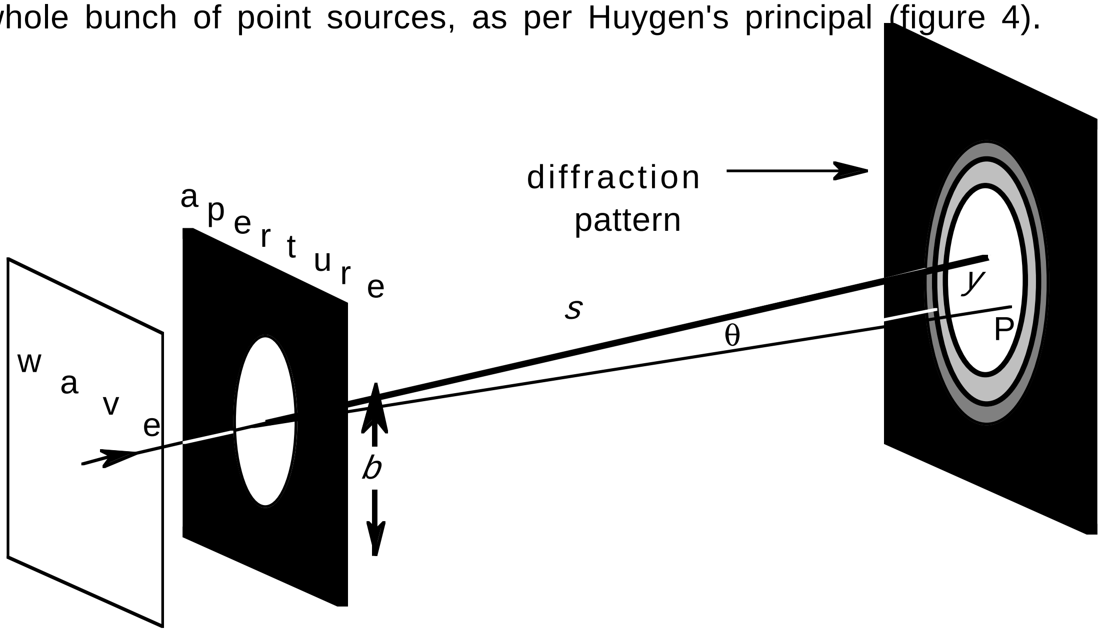


Figure 4. Fraunhofer diffraction by a circular aperture.

The mathematical problem is that of integrating over the area of the circular aperture to sum the contributions of all those tiny point sources to get the amplitude of the contribution at point P, the line to which makes an angle θ with the normal to aperture and screen. That amplitude is then squared, in the usual fashion, to get the illuminance of the pattern on the screen. The result is

$$E_\theta = 4E_o[J_1(\pi b \sin\theta/\lambda)/(\pi b \sin\theta/\lambda)]^2$$

This was derived in 1835 by Sir George Airy giving the characteristic pattern of figure 3. The function J_1 is a so-called "special function" known as a 1st order Bessel function. It can be calculated from the infinite series

$$J_1(x) = x/2 - (x/2)^3/(1!2!) + (x/2)^5/(2!3!) - \ldots$$

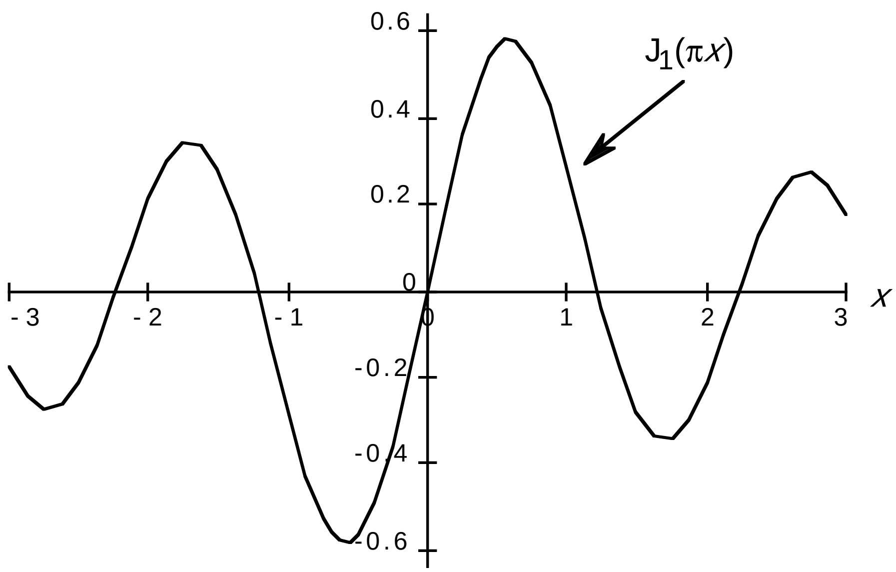


Figure 5. Graph of the Bessel function $J_1(\pi x)$.

These special functions are tabulated just like sines and cosines. As shown in figure 5, $J_1(\pi x)$ oscillates and has its first minimum at $\pi x=1.22$.

As can be seen from the equation above, the first minimum of the diffraction pattern occurs at that 1st zero in the Bessel function and using figure 5 we get so for the 1st minimum

$$\pi b\sin\theta/\lambda=1.22\pi$$

or

$$\sin\theta=1.22\lambda/b.$$

To get radius of the maximum, do as we did with single slit, noting that if y_1 is the radius

$$y_1=s\tan\theta\cong s\sin\theta=1.22s\lambda/b.$$

This central maximum is called the Airy disk. About 84% of the energy in the pattern is within the Airy disk.

Usually the pattern is seen focused with a lens of focal length f', the screen being at the secondary focal point F'. In that case substitute $s\Rightarrow f'$ in the equation above.

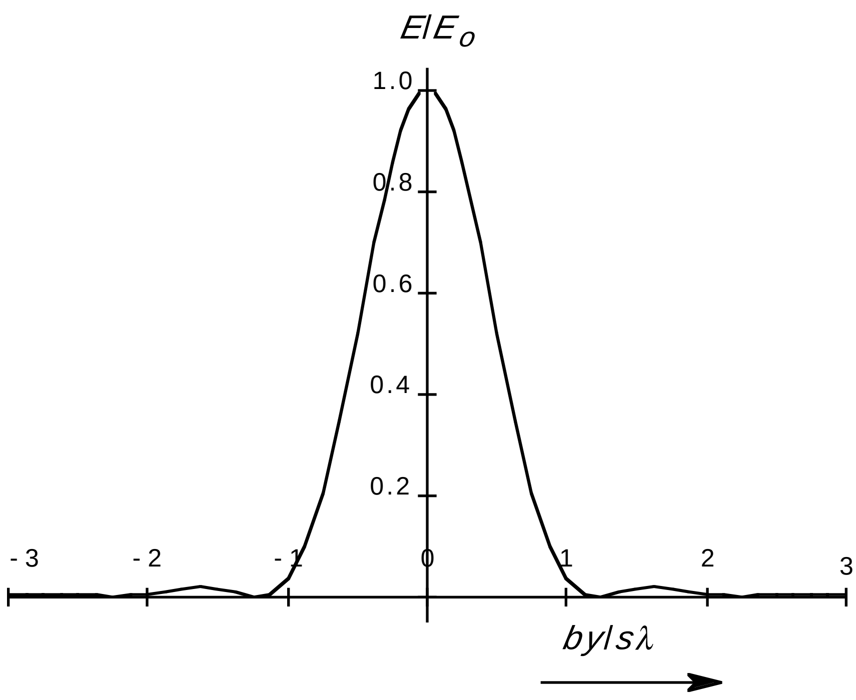
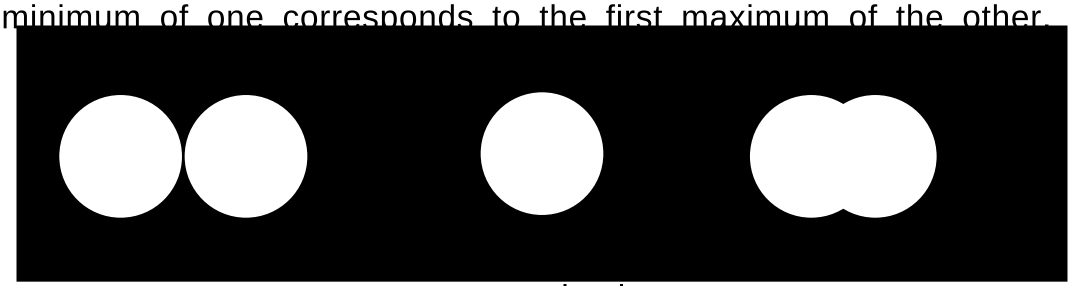


Figure 6. Illuminance distribution along the radius of the diffraction pattern due to diffraction by a circular aperture.

RESOLVING POWER

If the blur circles corresponding to the images of two point objects are sufficiently separated, the images are resolved. Rayleigh proposed a standard for the sufficient separation, that standard being the Rayleigh Criterion. The Rayleigh Criterion is that two point sources are just resolved if the Airy Disks corresponding to their images are such that the first minimum of one corresponds to the first maximum of the other.



two completely resolved images

two completely unresolved images

two images just resolved by the Rayleigh criterion

Figure 7. Images resolved and unresolved by the Rayleigh criterion.

It turns out that the images are just resolved if their angular separation is more than

$$\theta=1.22\lambda/b$$

where b is the diameter of the entrance pupil of the optical system.

Example: For the human eye, $b\approx 4$mm, so taking $\lambda=550$nm,

$$\theta=1.22\lambda/b=1.22\text{x}550\text{x}10^{-9}/(4\text{x}10^{-3})=1.7\text{x}10^{-4}\text{rad}$$

$$=9.6\text{x}10^{-3}\text{ deg}=0.6'\text{ of arc.}$$

Figure out the acuity by recalling that a 20 ft letter can be resolved by a patient who resolves 1' of arc. What letter can, then, be resolved by a patient capable of 0.6' of arc? Use proportions, as follows:

$$x/20\text{ft}=0.6'/1'\Rightarrow x=12\text{ft}.$$

Hence this corresponds to 20/12 acuity, not far from that of the average patient population.

Note from this example that any improvement in the geometric optics of the eye wouldn't improve resolution beyond what most patients have already. Neither would improvement of the retina by, say, packing receptors more densely. The eye is a diffraction limited system.

FRESNEL DIFFRACTION FROM A CIRCULAR APERTURE

In Fraunhofer diffraction, a collimated wave strikes an aperture and, after diffraction, proceeds to a remote receiving surface. In Fresnel diffraction either the incoming wave is uncollimated, the receiving surface is at a finite distance, or both. Experimentally it is much easier to produce Fresnel diffraction than Fraunhofer diffraction, but Fresnel diffraction is far harder to analyze.

Fresnel diffraction from a circular object or barrier produces a bull's-eye pattern somewhat like that of an Airy disk, but the width and brightness of the rings vary with the exact conditions. It's even possible to produce a pattern with a dark central ring.

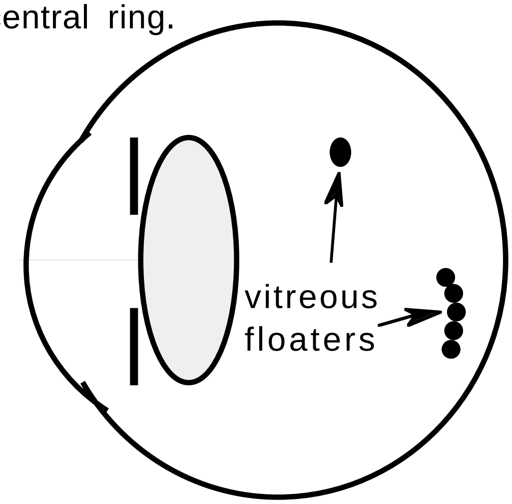


Figure 8. Vitreous floaters.

Fresnel diffraction explains the appearance of vitreous floaters, one of the most common visual complaints. Floaters are bits of protein, blood cells, etc., which come loose and float around in the vitreous casting their shadows on the retina. Sometimes they are small discrete objects, sometimes a chain of opacities. From geometric optics, one might expect the retinal shadows to be dark and sharp edged, but in fact patients typically refer to their floaters as being like "cobwebs" or "ripples in water".

The reason for this is that Fresnel diffraction prevents hard edged shadows, instead producing bull's-eye patterns or a shadow surrounded by ripples.

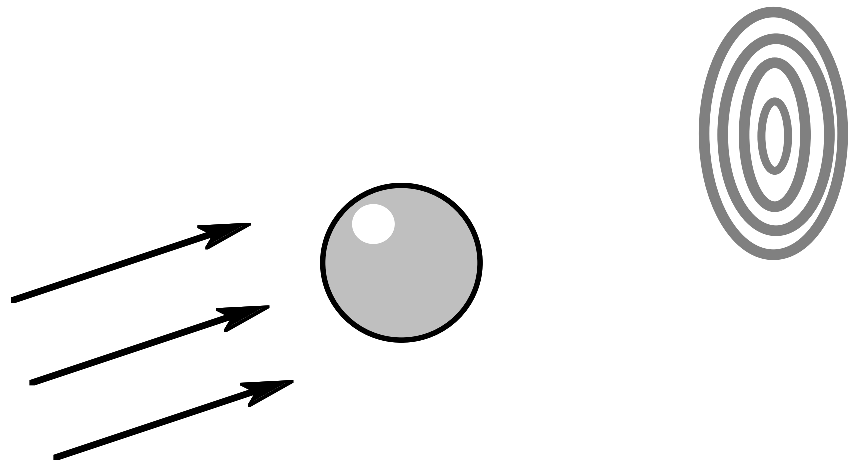


Figure 9. Fresnel diffraction around a spherical object produces the retinal pattern characteristic of vitreous floaters

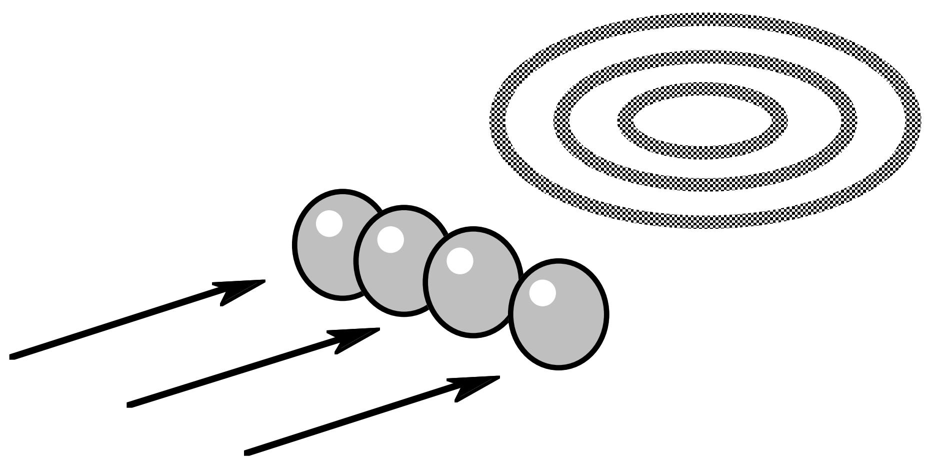


Figure 10. Diffraction by a chain of opacities produces the kind of ripply pattern patients often describe as being, "like a hair."

Fresnel's own analysis of Fresnel diffraction was based on dividing a wavefront up into zones, as in the diagram below. By Huygen's principle, each zone becomes a secondary source. The zones are constructed so that all the waves produced by the points in a zone are all positive, or all negative, when the wavefront arrives at some point P. In the diagram below, solid lines extending from a zone to point P indicate a positive amplitude contribution and dotted lines indicate a negative amplitude contribution.

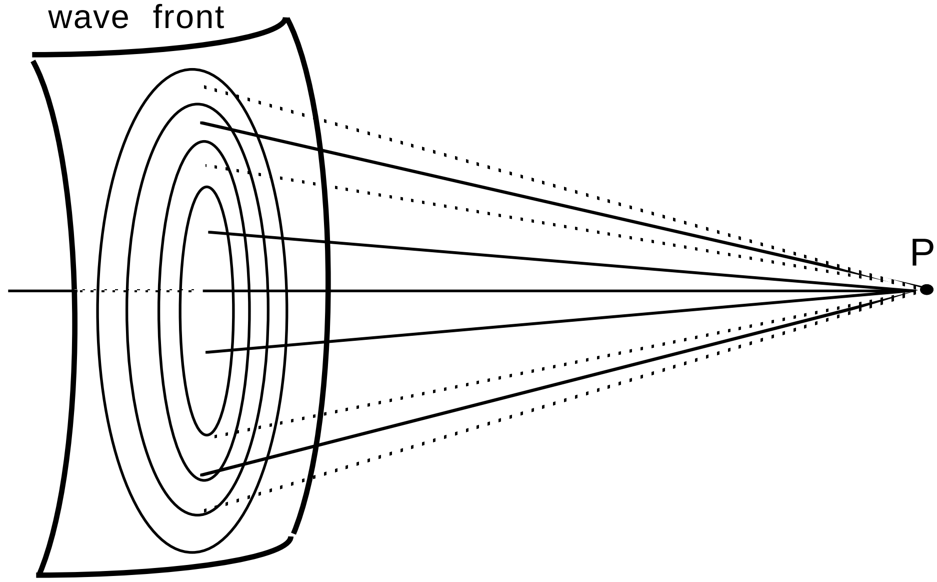


Figure 11. A wave front converging on point P.

Suppose we place a plate between the wave front and P so that the contributions from the negative zones are blocked off. This will increase the amplitude of the wave reaching point P, producing a bright spot. Such plates are called <u>zone plates</u>. A zone plate can dramatically increase the intensity of the light reaching P. Ten exposed zones, for example, can increase intensity 400 times.

Fresnel plates have lens-like properties. If, for example, the distance from a zone plate to a point source of light is l, the distance from the zone plate to the bright point P is l' where l and l' are related by the equation

$$1/l' = 1/l + 1/f'$$

where $f' = r_1^2/\lambda$ where r_1 is the radius of the first Fresnel zone and λ is the wavelength of the light passed through the zone plate.

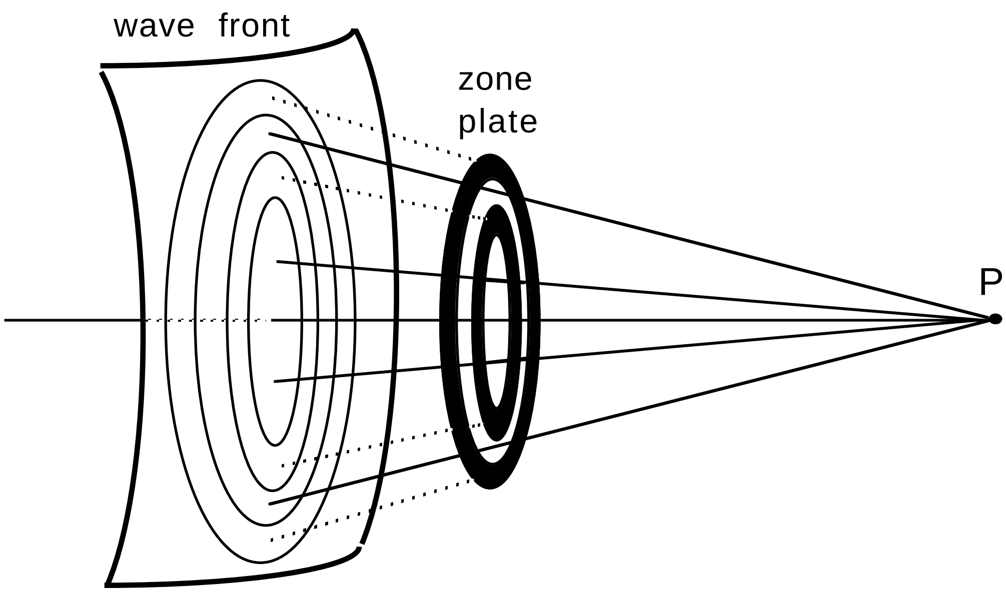


Figure 12. The Fresnel zone plate.

One manufacturer has attempted to exploit the lens-like properties of the zone plate in a soft contact lens bifocal. A zone plate replaces the conventional "add" lens used to view near objects.

DIFFRACTION WITH NON-CIRCULAR APERTURES

Diffraction patterns due to non-circular apertures preserve the general symmetry of the aperture, but with the longest dimension of the diffraction pattern corresponding to the narrowest dimension of the aperture. For example, the central maximum of the Fraunhofer diffraction pattern of a rectangular aperture is a rectangle with its widest dimension in the direction of the narrowest dimension of the aperture. Fraunhofer diffraction by an elliptical aperture produces an Airy disk-like pattern elongated along the minor axis of the ellipse (figure 13).

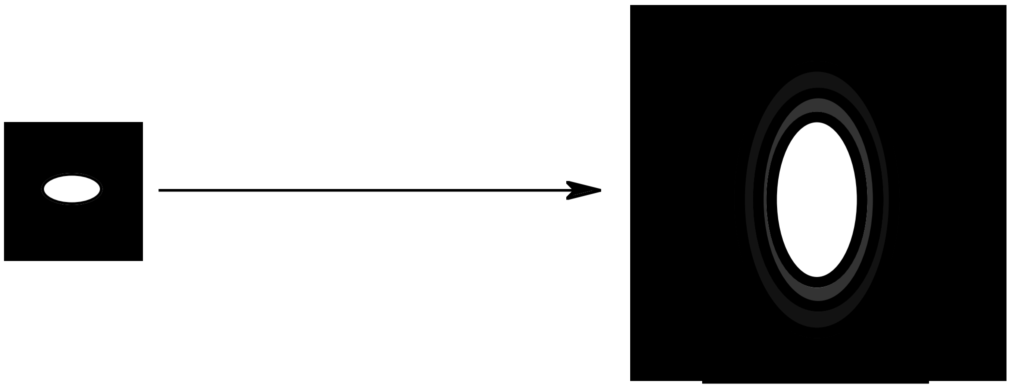


Figure 13. Diffraction by an elliptical aperture like that at the left produces the elliptical pattern at the right.