ELECTROMAGNETIC WAVES AND POLARIZATION

ELECTROMAGNETIC WAVES

Waves occur in a variety of physical contexts such as water waves produced by a stone dropped in a pond, w which travel along a plucked string, etc. In the last century James Clerk Maxwell assembled from the existing experimental and theoretical information available at the time a set of four partial differential equations governing the behavior of electromagnetic fields. In free space, using vector notation these equations (in cgs units) have the form

$$\nabla\cdot\boldsymbol{E} = 0$$
$$\nabla\cdot\boldsymbol{H} = 0$$
$$\nabla\mathrm{x}\boldsymbol{H} = (1/c)\partial\boldsymbol{E}/\partial t$$
$$\nabla\mathrm{x}\boldsymbol{E} = -(1/c)\partial\boldsymbol{H}/\partial t$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields, respectively, and c=3x10^{10}cm/sec.

A physics course in optics conventionally starts with this set of partial differential equations and derives all the properties of optics, e.g. Snell's law, from it. We won't do that here. Instead we'll summarize the results, as follows.

The solution to Maxwell's equations is of the form

$$E,\ H = \mathrm{f}(x\text{-}ct).$$

where f can be any function of a single variable. The electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are in phase with and perpendicular to one another. But we recognize this as the equation of a wave, hence the free field solution of Maxwell's equations is a travelling wave of speed c=3x10^{10}cm/sec. You may recognize that speed as the velocity of light in a vacuum.

In three dimensions the appearance of the wave (if we could see it) would be 2 perpendicular waves, one of electric field ***E*** and one of magnetic field ***H***, in phase, rippling along in a straight line (figure 1).

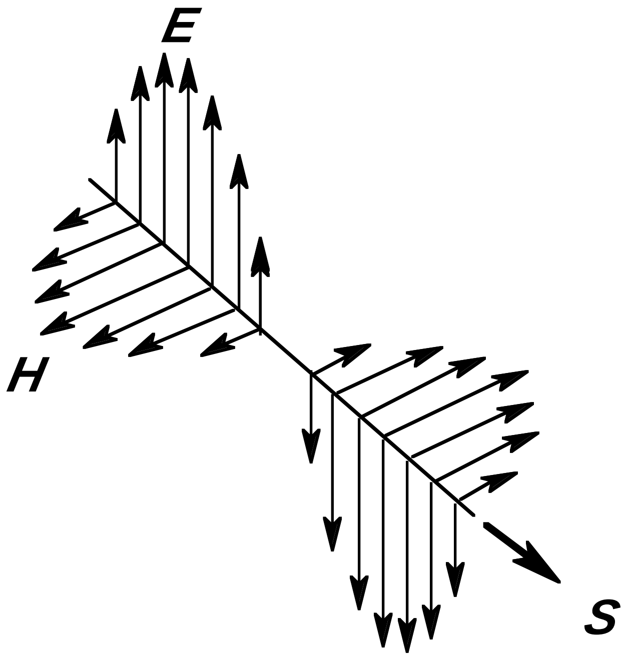

Figure 1. An electromagnetic wave has perpendicular oscillating electric and magnetic field vectors.

The energy carried by the electromagnetic wave is given by the Poynting vector, ***S***, where

$$\mathbf{S}=\mathbf{E}\times\mathbf{H}.$$

The energy carried by the wave is, therefore, proportional to the square of the amplitude of the wave. The energy density of an electromagnetic wave is given by

$$\text{energy per unit volume}=(E^2+H^2)/(8\pi)=E^2/(4\pi).$$

The important thing here is that the energy of the wave goes as the **square** of its amplitude.

ELECTROMAGNETIC SPECTRA

Electromagnetic waves can be radiated all across the spectrum. What we name a wave depends on what it "tickles". Waves producing vision (which tickle the retina) are light waves.

wavelength (meters)	
10^7	electric waves
10^6	
10^5	
10^4	radio
10^3	
10^2	television
10	
1	
10^{-1}	radar
10^{-2}	microwave
10^{-3}	
10^{-4}	
10^{-5}	infrared
10^{-6}	visible light
10^{-7}	ultraviolet
10^{-8}	x-rays

In the visible spectrum long wavelengths correspond to reddish colors and short wavelengths to bluish colors. Yellow is in the middle of the visible spectrum.

Different light sources produce different kinds of spectra, with different amounts of light energy at each wave length. Two main kinds are line spectra and continuous spectra.

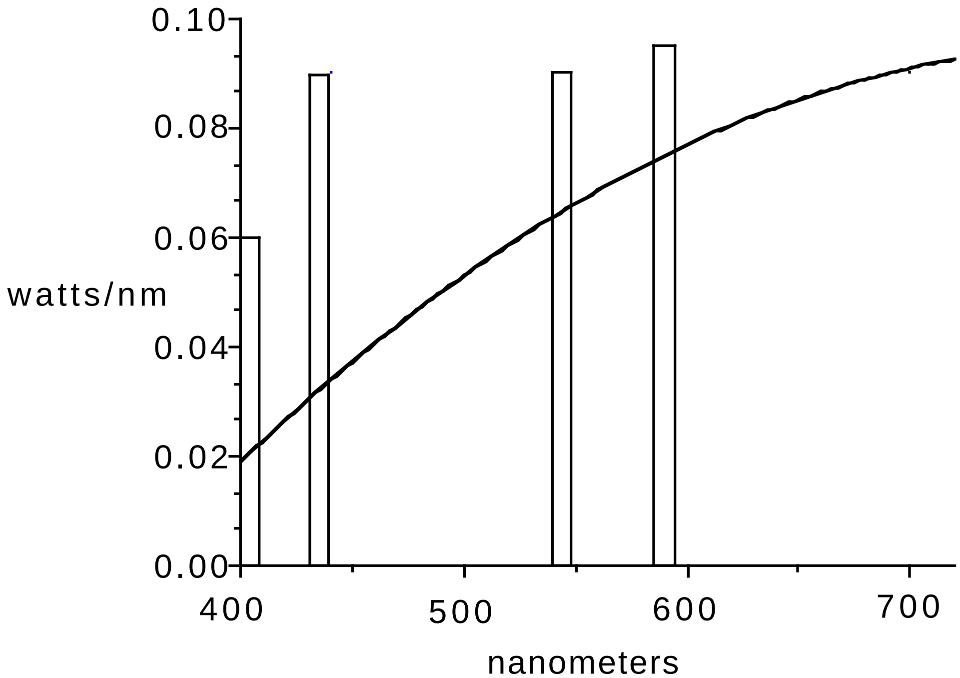

Figure 2. The smooth curve represents a continuous spectrum and the spikes are line spectra.

In figure 2 the continuous line is a portion of the so-called "black body" spectrum, which describes well the spectral distribution from a source like an ordinary tungsten light bulb. The spectral lines are those of a mercury vapor lamp which emits energy only at and around certain discrete wavelengths. Many light sources have a combination of continuous and line spectra.

POLARIZATION

Any beam of light of a given wavelength will contain many waves of light at random orientations. If we can produce a beam with only one orientation of ***E*** vectors (***H*** $\perp$ ***E***), the beam is polarized. If some mixture of other orientations exists, the beam is partially polarized.

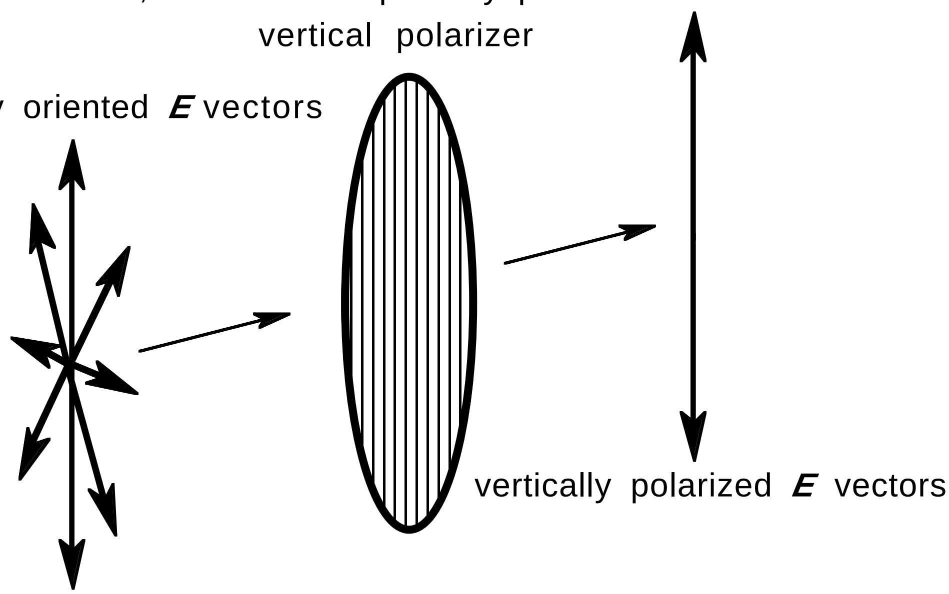

Figure 3. Unpolarized light passing through a vertical polarizer.

Consider two polarizers (of any type) which are placed with their polarization axes at an angle θ with one another. The electric vector of the wave passing through the first polarizer will, of course, make an angle θ with the polarization axis of the second polarizer. Only the component of the electric vector parallel to the second polarizer will be transmitted (figure 4).

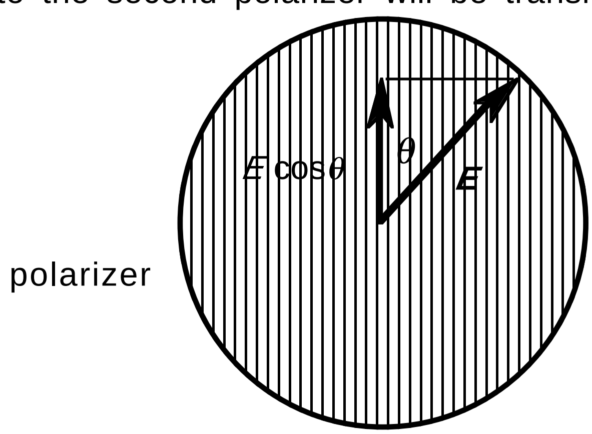

Figure 4. Only the vertical component of the electric vector of a wave is transmitted by a vertical polaroid filter.

If the electric vector has magnitude E, that component will be $E \cos\theta$. Since intensity is proportional to the square of amplitude, the apparent intensity of an object seen through a pair of crossed polaroids is given by Malus' law which is

$$I(\theta)=I_o\cos^2\theta$$

where I_o is the amount of light transmitted by the polarizer-analyzer pair when their axes are aligned and θ is the angle between the axes of the polaroids. Since cos(90°)=0, Malus' law states that no light gets through a polaroid pair with perpendicular axes.

In an ideal polaroid, there are no losses due to absorption, but in a real polaroid absorption must be added in when calculating the total light transmitted by crossed polaroids.

When an unpolarized beam of intensity I_o hits a polaroid, a beam of intensity $I_o/2$ is transmitted since the polaroid restricts light to that of a given polarization and that is half the ensemble of light waves striking the polaroid. For an unpolarized beam, then, a polaroid behaves like a neutral density filter with 50% transmittance.

METHODS OF POLARIZING A BEAM

REFLECTION

Rays may be polarized by bouncing a light beam off a dielectric, like glass or water.

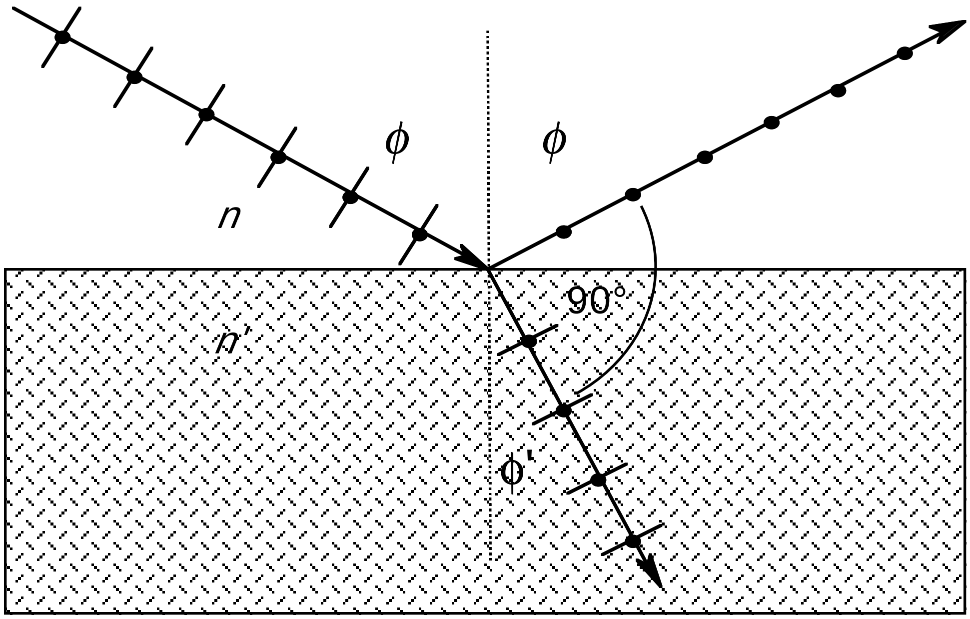

Figure 5. Complete polarization due to reflection. The dots represent polarization perpendicular to the plane of the page and the lines represent polarization in the page.

The exact laws governing polarization are Fresnel's laws of reflection. These show that greater and greater polarization occurs as one approaches the polarization angle (Brewster angle). For an interface between media of indices n' and n, that angle ϕ is $\tan\phi = n'/n$.

Another important result of Fresnel's law is that the amplitude of a wave reflected normally is given by

$$R=[(n-n')/(n+n')]E,$$

where E is the amplitude of the incident wave and R the amplitude of the reflected wave. Since $R<0$ if $n'>n$, there is a 180°=π radian phase shift in amplitude in reflection from rarer to denser media. There is no phase shift in going from denser to rarer media.

The fraction of energy reflected at an interface is proportional to the square of the ratio of amplitudes after and before reflection so the relative intensity of the reflected beam is

$$I/I_o=[(n-n')/(n+n')]^2.$$

DOUBLE REFRACTION

In certain crystals, e.g. calcite, a light beam acts as though there were two indices of refraction, each encountered by differently polarized rays.

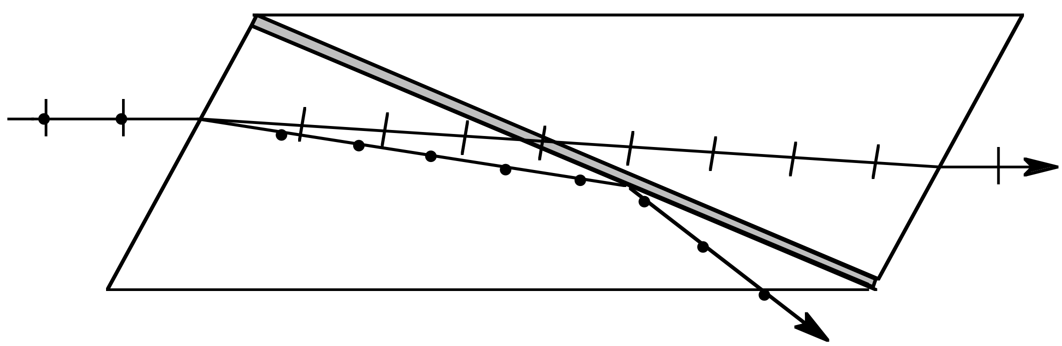

Figure 6. Optics of a Nicol prism. On entering the first calcite wedge of the prism horizontal and vertically polarized components see different refractive indices and are deviated in different directions. The one beam hits the Canada balsam layer at greater than the critical angle and leaves the prism. The other ray goes into the second calcite wedge and emerges undeviated but completely polarized.

This double refraction or birefringence is exploited in devices like the Nicol prism in order to obtain a polarized beam, as shown in figure 6.

Temporary double refraction or temporary birefringence is the production of double refraction in ordinarily singly refractive materials by stressing the material. This occurs in glass or celluloid. Temporary double refraction is exploited in studying stresses in architectural models or in checking for the Maltese cross pattern in heat treated ophthalmic lenses.

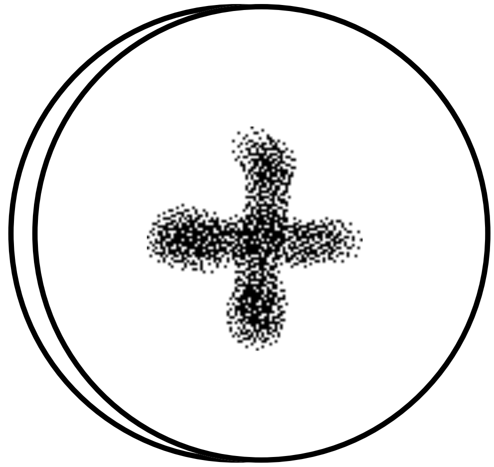

Figure 7. When placed between crossed polaroids, a heat tempered ophthalmic lens shows a characteristic Maltese cross pattern.

SCATTERING

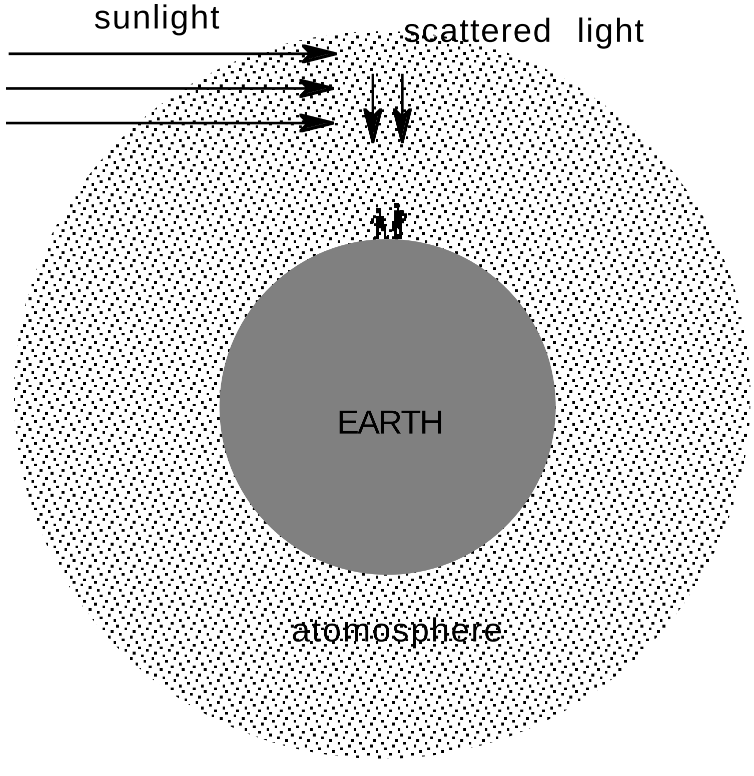

Figure 8. Rayleigh scattering of short wavelength light by the atmosphere produces partially polarized light when seen by an observer on earth.

Sunlight scattered by the atmosphere is somewhat polarized, the greatest polarization occurring when the sky is observed in a direction perpendicular to the sun's rays, as shown above. The polarization of the sky is only partial. The axis of polarization is generally vertical.

This is Rayleigh scattering, affecting mostly short wavelengths, hence the blue of the sky, the blue-white of fog or of the crystalline lens and cornea in the slit lamp beam. Since light scattered from fog is short wavelength, what color glasses should you prescribe for drivers who drive in fog?

SYNTHETIC POLAROIDS

Most common polarizers use synthetic polaroid sheets. Nowadays these consist of a special plastic (polyvinyl alcohol), chemically treated with iodine. The long molecules of the plastic are aligned by stretching the material during manufacture and serve as polarizing agents. Unlike Nicol prisms, polaroid sheets are cheap and allow large aperture polarized beams to be produced.

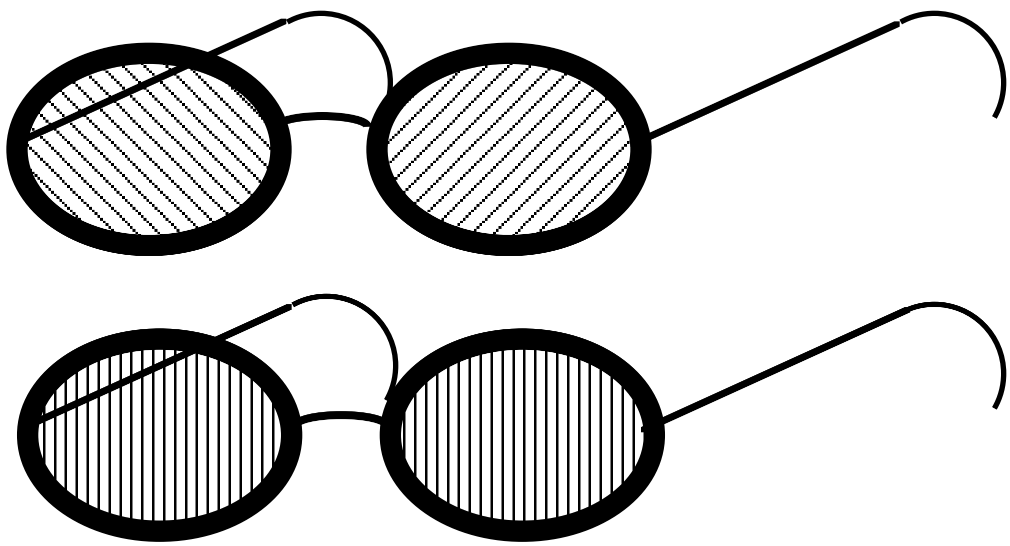

Figure 9. Two kinds of polaroid glasses. The polarization of the two lenses in the top pair are perpendicular to one another. These are used in viewing stereo targets. The polarization of both lenses in the bottom pair is vertical to cut out horizontally polarized glare from roads or water.

Synthetic polarizers sandwiched between layers of ophthalmic glass or plastic are used for polaroid sunglasses and stereo glasses. In polaroid sunglasses, the polarization plane is vertical to eliminate horizontally polarized glare. This is very useful to drivers to eliminate reflections from the road and boaters to eliminate reflections from the surface of the water. Anglers have discovered that polaroids will actually let them see *into* the water and there are polaroid sunglasses especially designed for fishermen. The lenses in stereo glasses are polarized 90° to one another. These give a different image to each eye. The images are then fused to give the illusion of depth.

HAIDINGER'S BRUSHES

Haidinger's brushes is an entoptic phenomenon in which an observer looking at a polarized beam of light will see a faint object which looks a little like a propeller or a shock of wheat. The phenomenon is thought to be related to radial polarization of Henle's fibers in the macula which act as the analyzer to the polarized light. Since the polarization is radial, the "brushes" will appear perpendicular to the orientation of the polarized beam. The phenomenon only occurs in the region of foveal fixation. Because it is faint and hard to observe, a rotating polaroid is often used to make the brushes spin and thus be more easily seen.

This latter fact has led to the use of Haidinger's brushes in pleoptics, a form of vision therapy used when a patient adopts a non-foveal fixation point on the retina with corresponding diminution of visual acuity. The brushes tag the direction of foveal fixation so the therapy centers around efforts to make the brushes and fixation point coincide. The brushes can also be used as a diagnostic technique to see how far the anomalous fixation point is from the fovea. The bull's-eye fixation target below is a typical configuration. Behind the target is a light source which shines through a rotating polaroid concentric with the target.

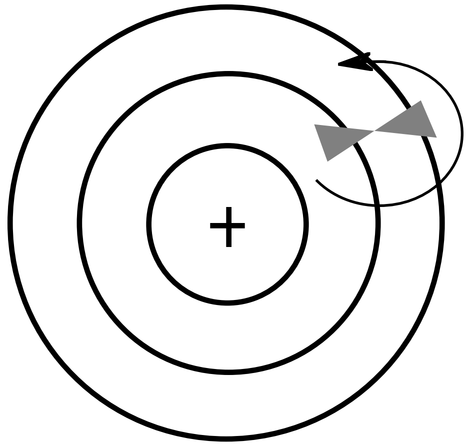

Figure 10. The center of this target is concentric to a polaroid filter rotating counter clockwise. When fixating the cross at an eccentrically fixating patient sees Haidinger's brushes above and to the right, indicating that the fovea is not being used for fixation.

RETARDATION PLATES

Suppose a plane polarized wave is sent through a birefringent material. If the plane of polarization is not parallel to the axes of the material, the horizontal and vertical components of the electric vector encounter different indices of refraction and so proceed through the material at different speeds. When the two components re-unite as the wave exits the material, the polarization of the wave will usually be changed, since the two components are now out of phase with one another.

Re-uniting these vertical and horizontal components produces, in general, elliptically polarized light in which the electric vector goes around in an ellipse instead of oscillating up and down as it does in plane polarized light. Since the electric vector of elliptically polarized light rotates through all angles, elliptically polarized light behaves like partially polarized light. In the figure 11 below, for example, the plane polarized light at the left will not pass through a horizontal polarizer while some light from the elliptically polarized light at the right will.

There are a few ophthalmic applications and implications of elliptically polarized light. One author, for example, has suggested that the cells of the corneal endothelium show up better in photographs made with circularly polarized light. Recently some orthoptists observed that the plastic prisms used in binocular vision therapy are birefringent. Plane polarized images from stereoscopic test plates became elliptically polarized when viewed through these prisms, so when patients observed polarized targets through the kind of polarized glasses shown above, some of the right eye image was exposed to the left eye and vice versa. This, of course, invalidated any test results.

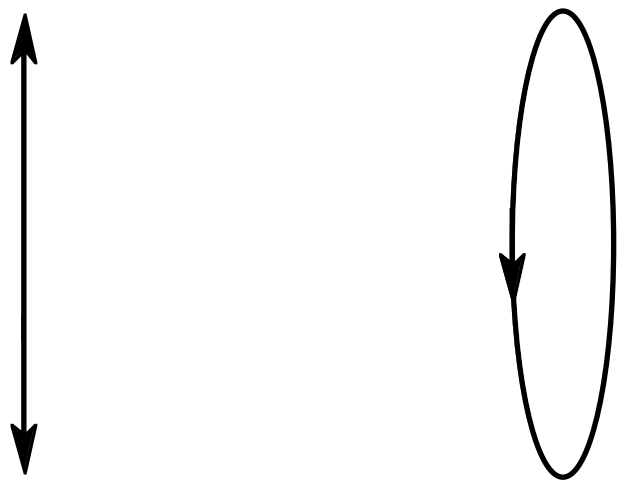

Figure 11. Trajectory of the electric vector in plane polarized light (left), and elliptically polarized light (right).

One special kind of retardation plate is the <u>quarter-wave plate</u> which has thickness $t=\lambda/(4|n_1-n_2|)$ where λ is the wavelength of light *in vacuo* and n_1 and n_2 are the two indices of the material. When plane polarized light is passed through a quarter wave plate, the plane of polarization is flipped around the optic axis of the material, as shown below.

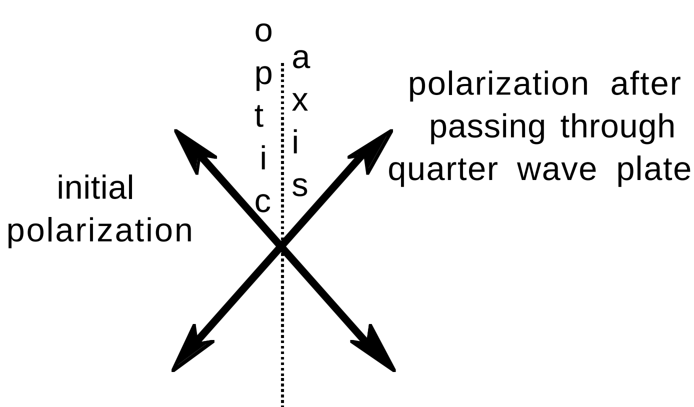

Figure 12. A quarter wave plate flips the polarization of plane polarized light around the plate's optic axis.

This has an interesting consequence in Haidinger's brushes. Assume Haidinger's brushes are produced by a polaroid filter rotating counter-clockwise. Four different positions of the brush pattern are shown in figure 13(a).

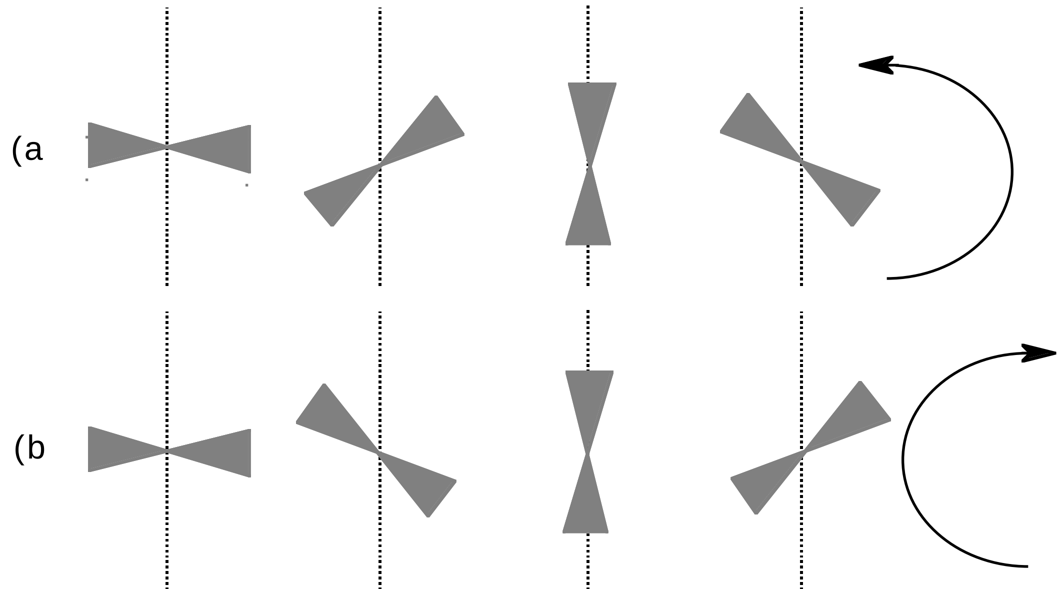

Figure 13. Four positions of Haidinger's brushes during rotation without a quarter wave plate (a) and with a quarter wave plot with vertical axis (b).

The corresponding orientations of the pattern after passing through a quarter wave plate with optic axis along the dotted line are shown in the figure 13(b). As is evident, the brushes now rotate clockwise. The quarter wave plate, therefore, reverses the direction of rotation of the brush pattern.

In clinical practice, pleoptics patients are invariably children and it is difficult to know if they're really seeing the hard-to-observe Haidinger's brush pattern. If the patient observes the change in direction of rotation of the pattern when he looks at it through a quarter wave plate, the doctor can be reasonably sure that the patient *is* , in fact, seeing the brushes. It turns out that cellophane is birefringent and, conveniently, the cellophane wrapping of an ordinary cigarette pack makes a good quarter wave plate.

OPTICAL ACTIVITY

In optical activity, the polarization of a polarized beam of light passing through an organic solution is rotated, the amount of rotation being greater with greater concentration of the solution. This provides a "dry" method for measuring the concentration of sugar in a solution, useful in the brewery business.