SLIT INTERFERENCE and DIFFRACTION

YOUNG'S SLITS

Young's double slits is the classic experiment in interference of light beams. In it a collimated wave front is incident on two very narrow slits of width much less than a wavelength of light, which then act as two separate but coherent light sources. The light goes to a screen where fringes can form, bright fringes where the two beams arrive in phase and dark fringes where the two beams arrive out of phase.

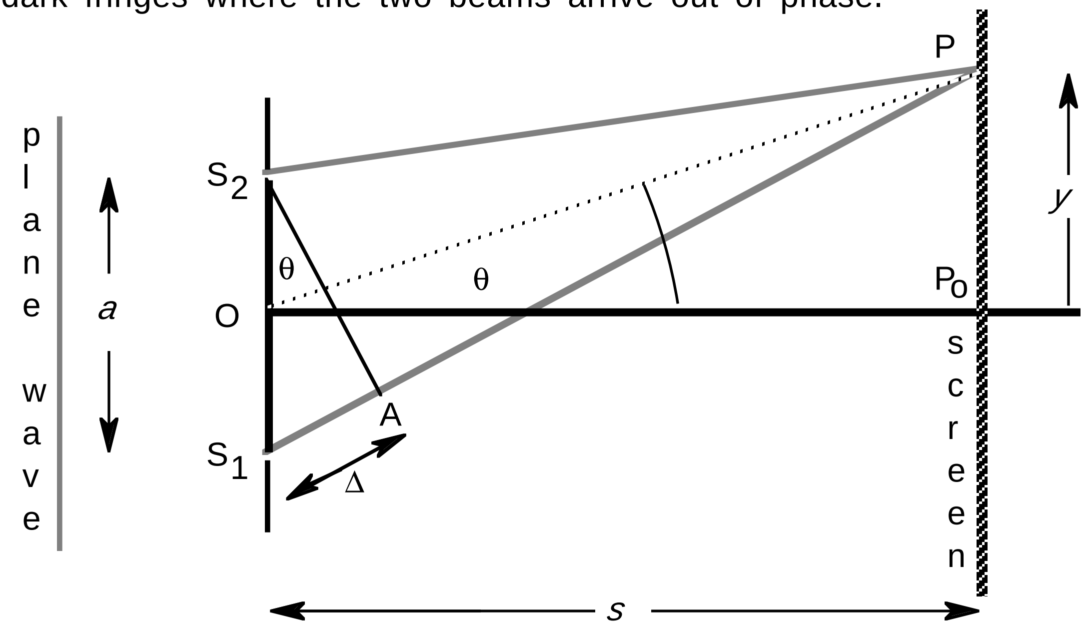

Figure 1. Young's double slits seen in cross section.

We'll get a bright fringe at P if $S_1P-S_2P=m\lambda$, where λ is the wavelength of the light incident on the slit. From figure 1, since $s >> a$,

$$\Delta=S_1P-S_2P=S_1A,$$

but

$$\Delta=a\sin\theta,$$

so a bright fringe occurs when

$$a\sin\theta=m\lambda.$$

But again from figure 1,

$$\sin\theta \cong P_oP/s = y/s,$$

and we get, finally,

$$ay/s = m\lambda.$$

or solving for y,

$$y = m\lambda s/a.$$

If the experiment is carried out in a medium of index n, $\lambda \Rightarrow \lambda/n$ and the preceding equation becomes,

$$y = m\lambda s/(na) \quad \text{[bright fringe condition]}.$$

Complete cancellation takes place halfway between the bright fringes where $S_1P - S_2P = (m+1/2)\lambda$ so that for dark fringes,

$$y = (m+1/2)\lambda s/(na) \quad \text{[dark fringe condition]}.$$

If the amplitude of the wave from each slit is E, the intensity at the bright fringes is proportional to E^2 and the intensity of the dark fringes is zero.

The next problem is to find the intensity at an arbitrary point between bright and dark fringes, thus getting the intensity distribution on the screen. This will be proportional to the square of the amplitude of the sum of the two waves when they arrive at point P.

The two waves arriving at P have amplitudes

$$\Psi_1 = A\sin[k(S_1P) - \omega t],$$

$$\Psi_2 = A\sin[k(S_2P) - \omega t].$$

These two waves differ by a phase factor δ where

$$\delta = k(S_1P - S_2P) = k\Delta = (2\pi/\lambda)\Delta = (2\pi/\lambda)(a\sin\theta) = 2\pi ay/(\lambda s).$$

For two waves Ψ_1 and Ψ_2 such that

$$\Psi_2 = A\sin(kS_2P - \omega t),$$

$$\Psi_1 = A\sin(kS_2P - \omega t + \delta),$$

the sum is

$$\Psi = \Psi_1 + \Psi_2 = A[\sin(kS_2P - \omega t) + \sin(kS_2P - \omega t + \delta)].$$

The fact that these are of equal amplitude permits certain simplifications.

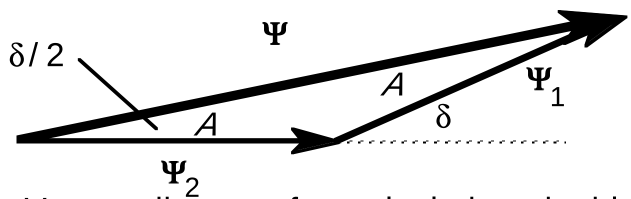

Figure 2. Vector diagram for calculating double slit interference amplitude.

From the vector diagram of figure 2, the phase factor of the resultant is, clearly, $\delta/2$. The magnitude of the resultant is just

$$|\Psi| = \sqrt{\{[A\sin\delta]^2 + [(A + A\cos\delta)]^2\}}$$

or, simplifying,

$$|\Psi| = \sqrt{\{A^2[\sin^2\delta + \cos^2\delta] + 2A^2\cos\delta + A^2\}} = \sqrt{\{2A^2(1+\cos\delta)\}}.$$

Using a general trigonometric identity $1+\cos\delta = 2\cos^2(\delta/2)$ so that,

$$|\Psi| = \sqrt{\{4A^2\cos^2(\delta/2)\}} = 2A\cos(\delta/2).$$

Hence Ψ is

$$\Psi = 2A\cos(\delta/2)\sin(kS_2P - \omega t + \delta/2).$$

We can apply this last result directly to the Young's double slits problem. The square of the amplitude of the wave at P is given by

$$\text{amplitude}^2 \propto 4A^2\cos^2(\delta/2) \propto A^2\cos^2[(\pi a y)/(\lambda s)].$$

The illuminance (the amount of light flux per unit area) at a point y off the midline is then given by

$$E \propto \cos^2[(\pi a y)/(\lambda s)].$$

Letting the proportionality constant be E_o,

$$E = E_o \cos^2[(\pi a y)/(\lambda s)]. \tag{1}$$

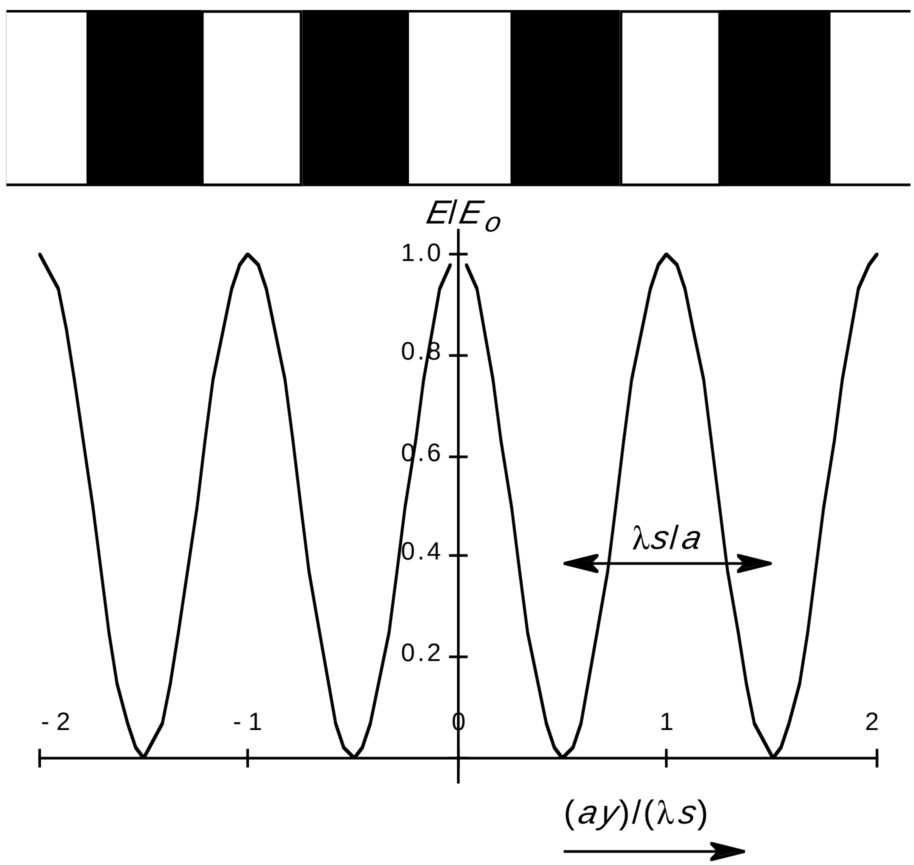

Figure 3. Illuminance distribution of the double slit fringe pattern. The corresponding fringe pattern formed on the screen is represented at the top.

Equation (1) is plotted in figure 3. Corresponding to this illuminance distribution is a pattern of bright and dark stripes or <u>fringes</u> on the receiving screen. The peaks of the illuminance distribution correspond to the center of the bright fringes and the valleys correspond to the center of the dark fringes. The illuminance gradually changes between the bright and dark fringes, although this is not represented in figure 3.

Practical application of the double slit interference phenomenon may be made by evaluating retinal integrity in cataract patients by checking their sensitivity to fringe patterns formed directly on the retina by an instrument like the Rodenstock retinometer.

Suppose we had, instead of a double slit, a triple slit. As the diagram shows, there are now three waves contributing to the intensity pattern. The phase difference between the wave from S_3 and S_2 is the same as the phase difference between S_1 and S_2 in the double slit case.

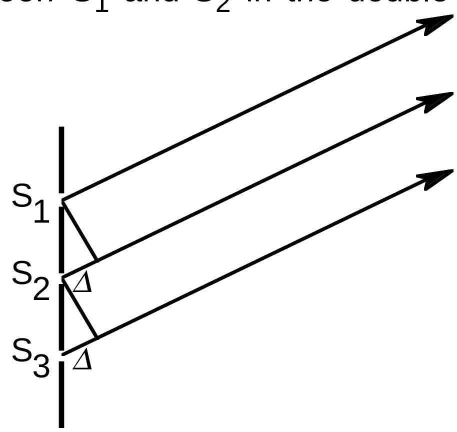

Figure 4. Young's triple slits.

The intensity at a point P on the screen is calculated by adding three vectors.

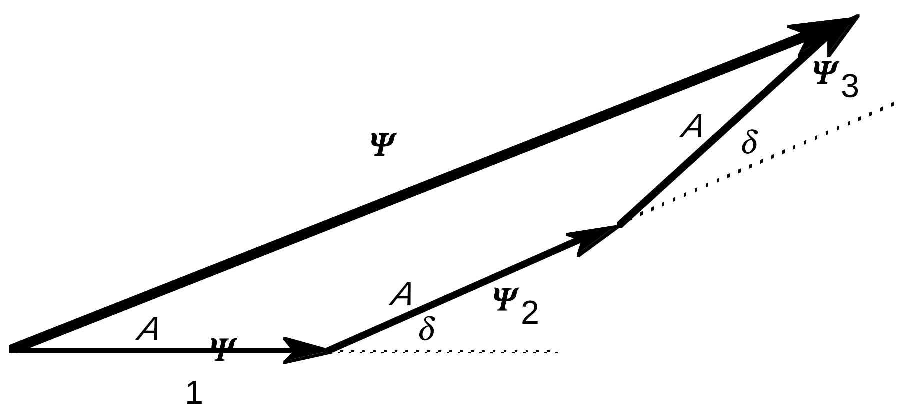

Figure 5. Vector addition of amplitudes for Young's triple slits.

The intensity distribution is similar to that of the double slit, but there are additional secondary maxima crammed between the primary fringes which would make them narrower.

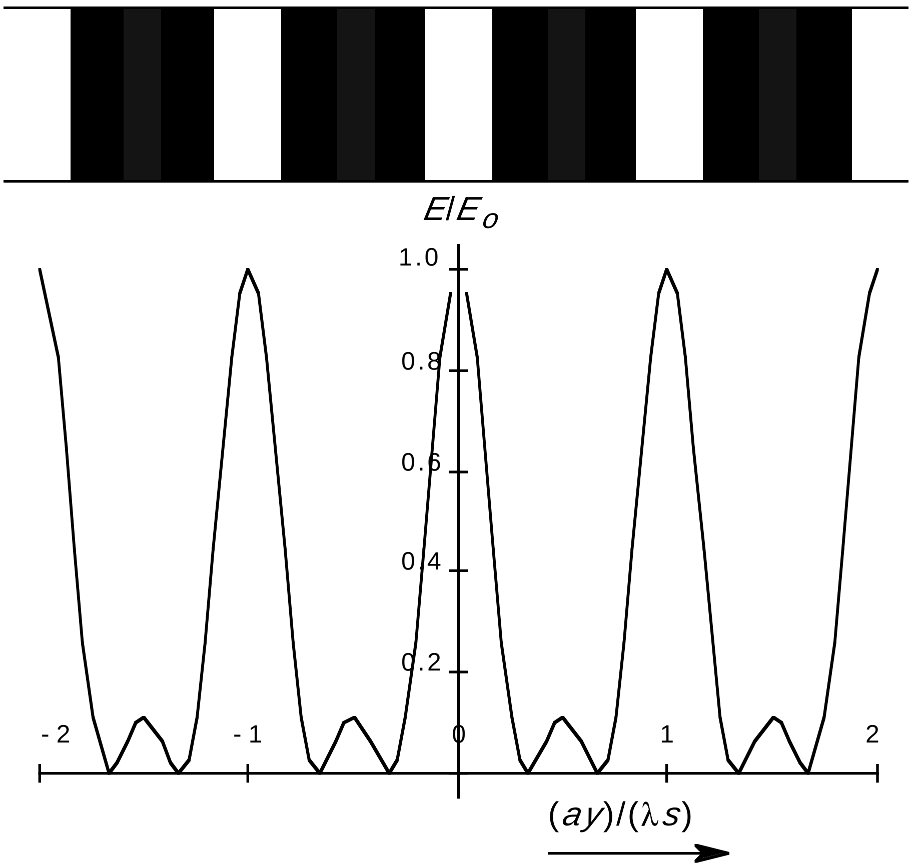

Figure 6. Illuminance distribution of the triple slit fringe pattern. The corresponding fringe pattern formed on the screen is represented at the top.

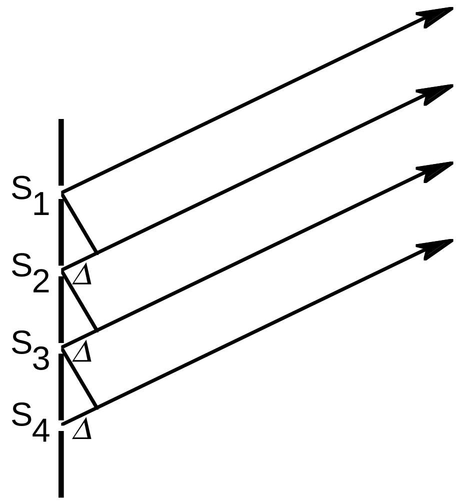

Figure 7. Young's quadruple slits.

For quadruple slits four waves contribute, the wave from each slit having the same phase difference from the one above. The amplitude is calculated by adding four vectors as shown in figure 8.

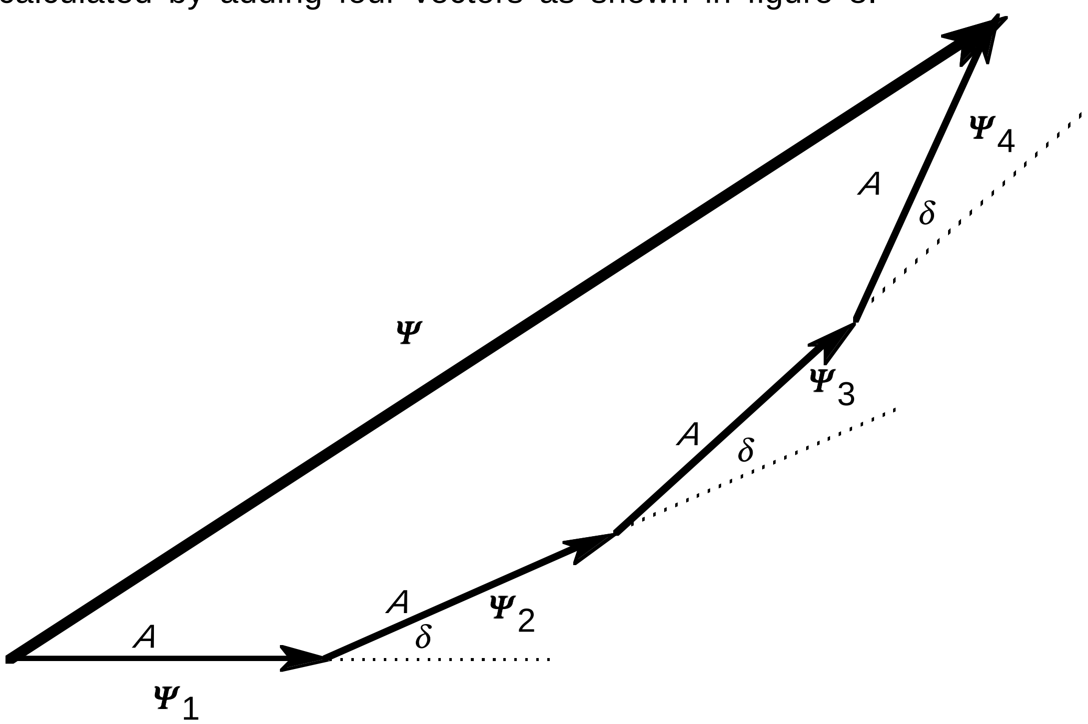

Figure 8. Vector addition of amplitudes for Young's quadruple slits.

The corresponding intensity distribution now has even more sharply defined peaks, this time with two secondary maxima between the primary maxima. In general, the number of secondary maxima between the primary maxima is two less than the number of slits.

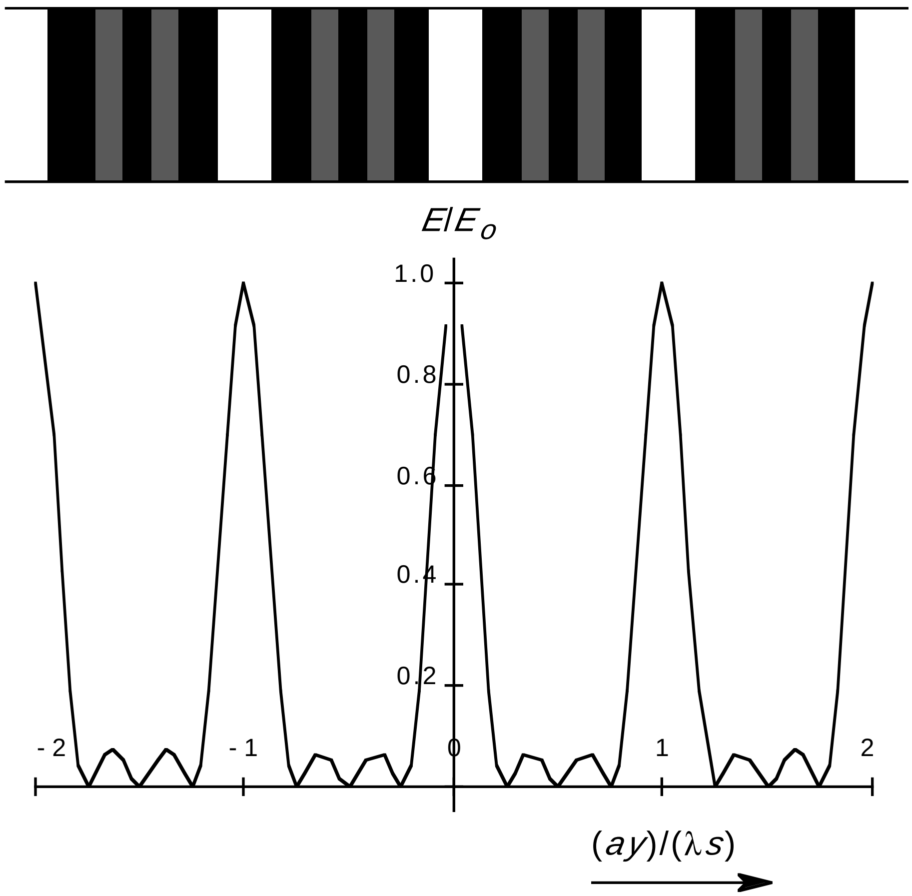

Figure 9. Illuminance distribution of the quadruple slit fringe pattern. The corresponding fringe pattern formed on the screen is represented at the top.

As more and more slits are added, the primary maxima become brighter and more narrow as the secondary maxima become more numerous and less visible. For a monochromatic source, this produces line spectra like that of figure 10.

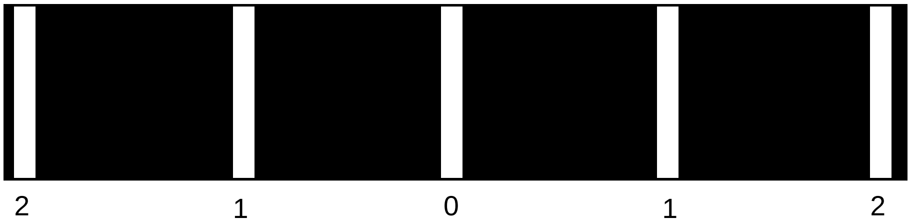

Figure 10. Fringe pattern formed on the screen with multiple slits.

This is the spectrum produced by a grating. The lines are numbered by their order. The larger the wavelength of light sent through a grating, the greater the separation of the orders.

Suppose a light beam with two wavelengths is sent through a grating. The resultant spectrum will look like that below. As the figure shows, the higher the order, the greater the separation of the lines corresponding to different wavelengths.

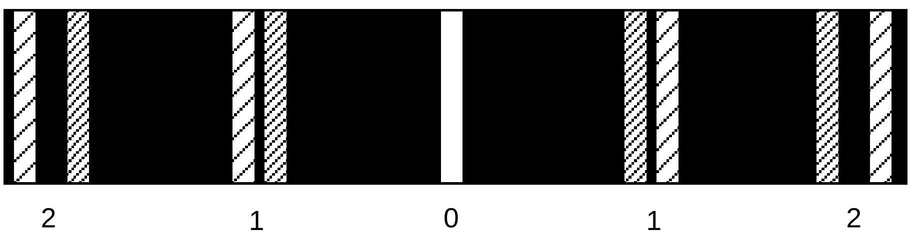

Figure 11. Fringe pattern formed on the screen when light from two monochromatic sources passes through multiple slits. The darker shading indicates fringes from the shorter wavelength.

Gratings may be either the transmission type we've discussed above where light is passed through an array of closely spaced lines or the reflection type in which light is reflected from a surface covered with finely ruled parallel lines. Gratings have a number of practical applications, especially in monochromators. These are used in analyzing the chemical content of vapors—terrestrial or galactic, in preparing truly monochromatic targets for color vision experiments, etc. Inexpensive gratings have been used to provide colorful lettering for house numbers, signs, and the like. Gratings even show up naturally on the backs of several species of beetles where they probably provide a signal confusing to predators.

SLIT DIFFRACTION

Now consider the **single** slit problem, but with a difference: the slit has a width *b* which is **not** much smaller than a wavelength of light. If the beam is collimated, geometric optics predicts that this single slit will cast a slit-shaped shadow its own size on a receiving screen. Physical optics predicts something rather different, as we'll see.

To solve the single slit diffraction problem, pretend the finite-width single slit is made up of a large number (infinite, really) of very small (infinitesimal) slits, each side by side. The amplitude at point P on a screen may be determined by summing the contributions of each of these tiny slits. The contribution from each slit is infinitesimal, but since there is an infinite number of them, the resulting contribution is finite. (This sounds like a problem for what type of mathematics?)

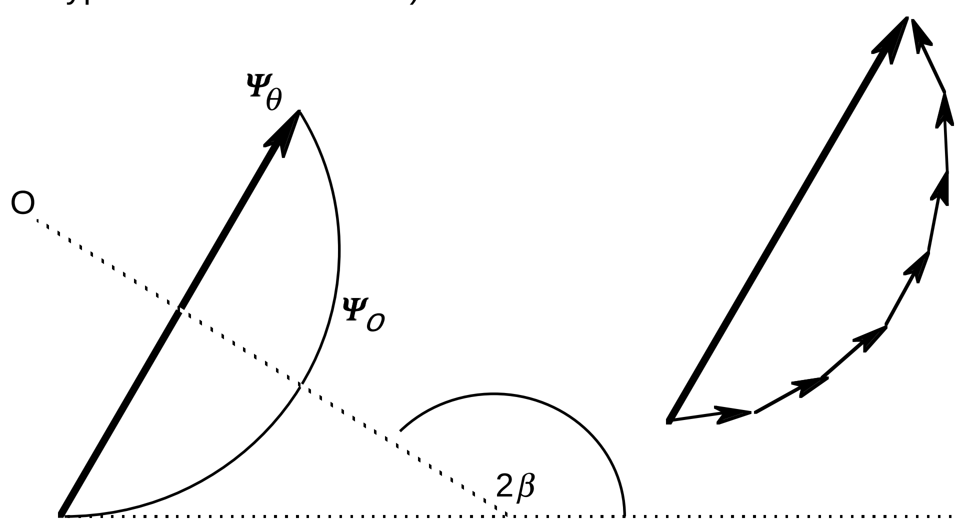

Figure 12. If we break a slit up into seven small slits and add their contributions, we get the diagram above right. As we increase the number of component slits, less light goes through each slit and the corresponding vector becomes shorter. At the same time the phase difference between adjacent sub-slits gets smaller. As the finite slit is broken down into more and more sub-slits, the vector diagram approaches the figure above left with its smooth circular arc.

Let the width of the slit be b and let there be N subslits so each subslit has width b/N. From the diagram, the phase difference between adjacent sub-slits is

$$\delta = k\Delta = (kb/N)\sin\theta = (2\pi/\lambda)(b/N)\sin\theta,$$

where we've used the same kind of reasoning we did with Young's double slits. The phase difference between the top sub-slit and the bottom one is just

$$N\delta = Nk\Delta = kb\sin\theta = (2\pi b/\lambda)\sin\theta \equiv 2\beta.$$

This allows us to label our graph as shown in the diagram above. Here the arc represents the maximum amplitude Ψ_o and the straight line chord represents Ψ_θ, the amplitude corresponding to a specific direction θ. Clearly, the diagram will be different for each θ, the arc length staying constant while the chord varies.

Working through this geometry gives the relationship,

$$\Psi_\theta = \Psi_o \sin\beta/\beta\ .$$

Squaring to get the intensity of the pattern,

$$I_\theta = I_o(\sin\beta/\beta)^2$$

where I_o is the intensity of the pattern at its center.

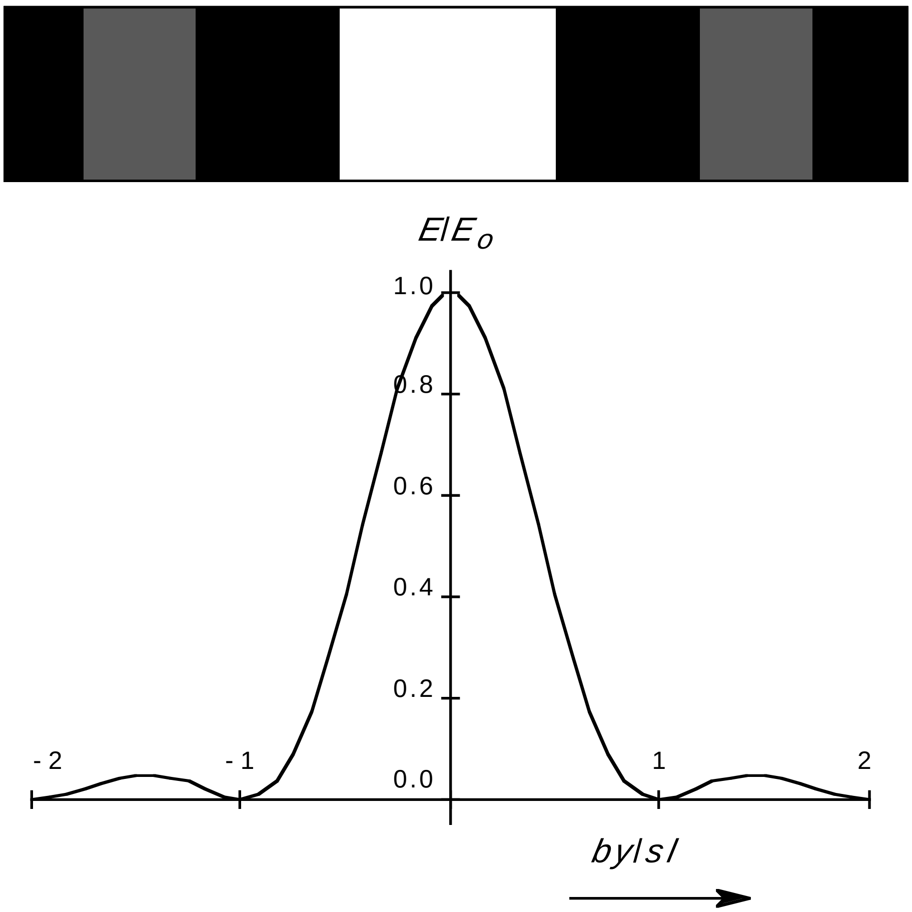

Figure 9. Illuminance distribution of the single slit fringe pattern. The corresponding fringe pattern formed on the screen is represented at the top.

Figure 9 shows the illuminance distribution for single slit diffraction. The plot shows a broad central maximum. Most of the energy is in this central maximum. The function I_θ has its zeros at

$$(\pi b/\lambda)\sin\theta=\pm m\pi$$

or

$$(b\sin\theta)/\theta=\pm m.$$

Since $\sin\theta=y/s$ the width of the central maximum is given by

$$\text{width of central maximum}=2s\lambda/b$$

From this latter equation it may be seen that the width of the maximum increases with decreasing b or increasing λ or s.

As more of slits of equal width are added to the system, we get a Young's multiple slit pattern modulated by a Fraunhofer envelope. The Fraunhofer envelope is that which would result with just one of the slits. The case for double slits is shown in figure 10.

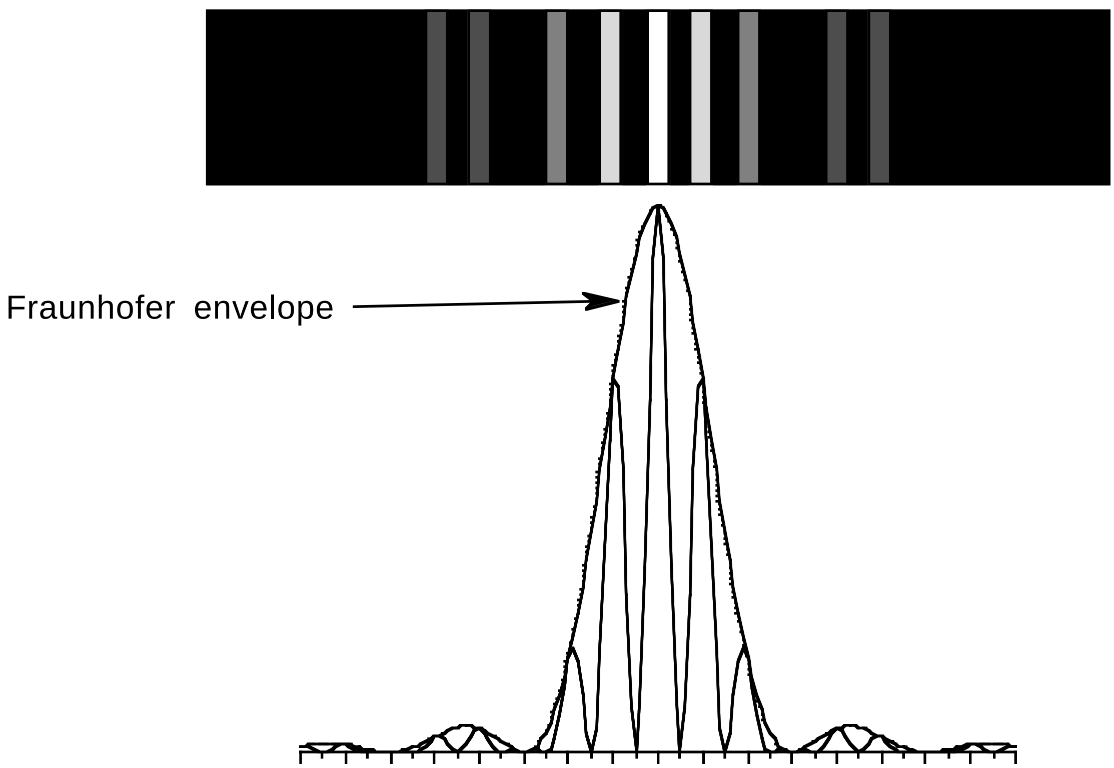

Figure 10. Illuminance distribution with interference caused by light diffracted by two slits of equal, finite width. The corresponding fringe pattern formed on the screen is represented at the top. Note how the Young's double slits interference pattern is modulated by the Fraunhofer envelope.

Note all the above considerations were for a collimated beam (from an optically remote source) striking a slit and proceding to an optically remote receiving screen. If either source or screen is at a finite distance from the slits, we're dealing with Fresnel diffraction, a much more difficult mathematical matter.

SUMMARY

The theory of slit interference and diffraction involves lots of equations which look pretty similar. The most important ones for working problems are summarized in the table below.

Young's Double Slits

Collimated coherent light of wavelength λ passes through two very narrow slits separated by distance a to a screen a distance s from the slits:

separation of bright fringes = separation of dark fringes = $\lambda s/a$

Single Slit Diffraction

Collimated coherent light of wavelength λ passes through one slit of width b to a screen a distance s from the slits:

width of central maximum = $2\lambda s/b$

Multiple Slit Diffraction

Collimated coherent light of wavelength λ passes through N slits of width b separated by distance a to a screen a distance s from the slits:

width of central maximum of Fraunhofer envelope = $2\lambda s/b$

separation of primary maxima = $\lambda s/a$

width of primary maxima = $2\lambda s/(aN)$

width of secondary maxima = $\lambda s/(aN)$

number of secondary maxima between primary maxima = N-2